JOURNAL OF APPLIED PHYSICS

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Lightning Ripping Its Way Through a Transmission Pole

JOURNAL OF APPLIED
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OCTOBER, 1939 Vol. 10, No. 10

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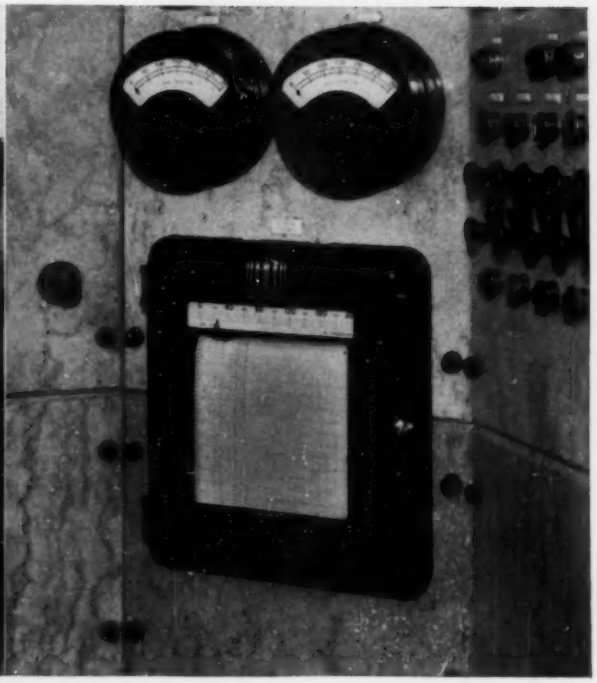

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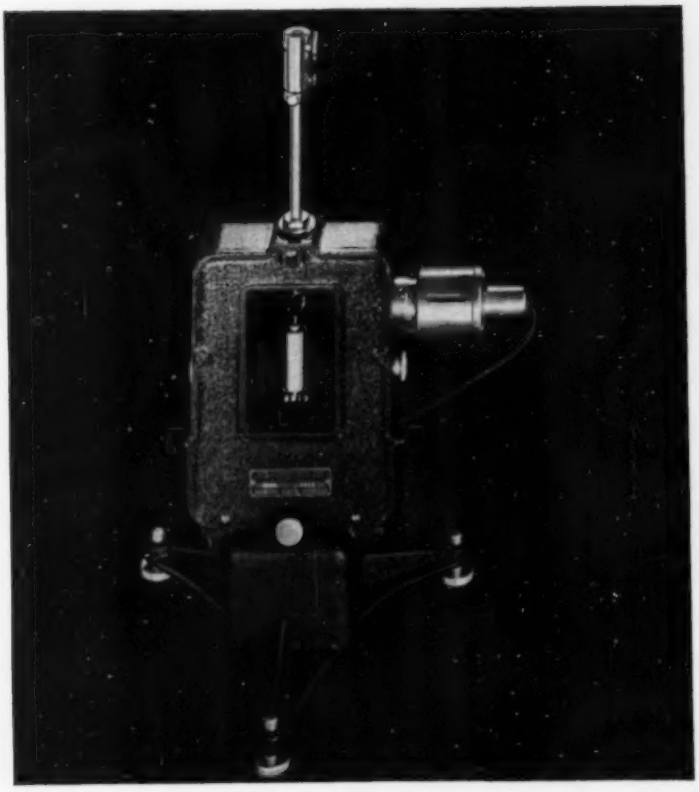

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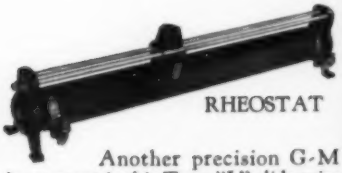

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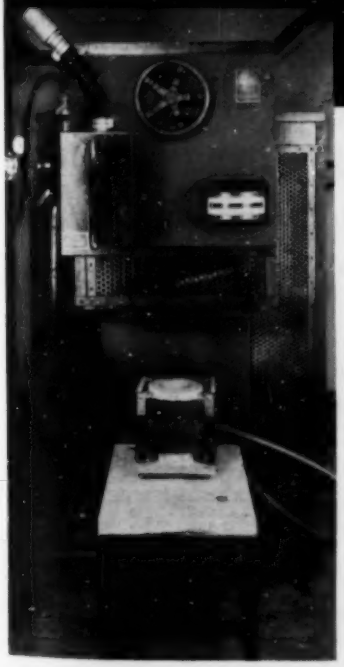

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"The World of Creative Physics"

THE writing of a popular book on applied physics was one of the first recommendations of the Advisory Council on Applied Physics when it was formed in 1935. It was hoped that a book could be written which would do as much for physics as Slosson's *Creative Chemistry* has done for chemistry. Dr. George Harrison, director of the George Eastman Research Laboratories of Physics, at Massachusetts Institute of Technology was asked to undertake the task of writing this book. In brief, that is the background of Dr. Harrison's new book *Atoms in Action—The World of Creative Physics** which has just been published.

Dr. Harrison has done a remarkably fine job. He has a clear, simple and straightforward style that will please both scientist and lay reader. One will chuckle with delight when he reads about the physicist "backing a bit of energy into a corner" so that "he can pounce on it in a new guise," or the study of "penguin-like" photons, or the disintegration of atoms in which the "atomic debris comes flying out like dirt from a gopher hole in which a very industrious puppy is scratching." The high literary quality of this book is attested by the fact that certain chapters were preprinted in modified form in *The Atlantic Monthly* and *Harper's Magazine*. One cannot help but admire the chapter headings such as "When Physics Goes Farming," "Glass—More Precious than Rubies," "Sound Rides the Wires," "Outwitting the Weather," and "The End is Not Yet."

Only those who have followed the writing of this book from the beginning have any idea of the enormous amount of energy, travel, and patience that have gone into its preparation. Some of the chapters have been written and rewritten a dozen times, each time in a clearer and more interesting manner. Dr. Harrison's bubbling enthusiasm is apparent on each page. The closing paragraph is a fine sample of the style that prevails throughout the book.

"Whether the world of the future derives its motive power from sunbeams, from atomic rays, or merely from the refinement of present sources of power, it will be an interesting world. Man holds within his hands the power to make that world virtually what he will. In particular, one of the bits of wisdom he is slowly learning in his gradual climb is that nature need not be accepted as she is at the moment, but can be smoothed out over a period of time and over an area of space, to make the ups and downs of life less severe."

The book is now finished and is off the press. Let us hope that it does for physics as a whole what the Advisory Council and the officers of the American Institute of Physics expect it to do. Dr. Harrison has done more than his bit. It behooves the rest of us to do everything possible to see that the book is widely publicized.

* William Morrow and Company, Inc., New York, 1939, pp. 371+x, Figs. 19, 14½×21½ cm. Price $3.50.

The Properties of Oxide-Coated Cathodes. I

BY JOHN P. BLEWETT

General Electric Research Laboratory, Schenectady, New York

1. Introduction

MORE than a third of a century has elapsed since Wehnelt[1] published his observation (1904) that a filament coated with an alkaline earth oxide is a copious low temperature source of electrons. The technical importance of this discovery has resulted in a continuous stream of publications; yet, in spite of a great deal of painstaking experiment, the mechanism which leads to activation of an oxide-coated cathode is still very incompletely understood. The influence of unforeseen factors has led to the accumulation of a mass of conflicting evidence and to the general conclusion that the mechanism of activation of an oxide-coated cathode is a complex one.

We shall deal for the most part with evidence obtained during the last decade. Several surveys have appeared during that time, the most comprehensive being those of Schottky, Rothe and Simon,[7] Dushman,[9] Gehrts,[11] Reimann,[13] and de Boer.[14] Representative references to earlier work will be found in these papers.

The present article will begin with a brief resumé of oxide-coated filament technique and will then proceed to a discussion of the various details involved.

In this paper, instead of writing "barium" to mean "barium and/or strontium and/or calcium," as has been customary in the past, we shall use the symbol (BaSrCa) to mean "barium *or* strontium *or* calcium." Similarly, (BaSr) will mean "barium or strontium," and so forth. Thus we shall avoid implying that experimental conclusions apply to cases where they have not been tested.

Brief summaries will be found in Tables I and II (p. 669) of those properties of (BaSrCa) and their oxides which are of particular interest in connection with oxide cathodes.

2. Resumé of Oxide Cathode Technique

Since the alkaline earth oxides are unstable in the presence of the gases of the atmosphere, oxide-coated cathodes are usually prepared by coating a metallic base with the alkaline earth carbonate, nitrate or hydroxide. This cathode is then heated in a vacuum to reduce the coating to the oxide. The gas evolved is pumped away and the cathode is then "activated." The activation process may consist in further heating, application of anode voltage to draw electron current, bombardment in a gas discharge, or some similar procedure—the end result being a cathode which emits electrons as copiously at 1000°K as does a tungsten filament at about 2300°K. The emission of an ordinary oxide cathode is shown as a function of temperature in Fig. 1, which includes for comparison the corresponding graphs for tungsten, thoriated tungsten, and thoria-coated cathodes.

The low operating temperature of oxide cathodes is coupled with high efficiency of electron emission. An oxide cathode running at 1000°K will emit about 100 ma per sq. cm with an efficiency of about 20 ma per watt input. A tungsten filament at 2300°K will give about the same emission, but its efficiency will be less than 1 ma per watt input. The consequent "low cost of electrons" from oxide cathodes has led to their very general use in commercial vacuum tubes. Oxide coatings are particularly applicable in cases where the cathode must be an equipotential surface and so cannot be heated by direct passage of current. The cathode then commonly takes the indirectly heated form, which is merely a small externally coated can heated from inside by an insulated tungsten filament.

The useful life of an oxide cathode operated at 1000°K should be several thousand hours. At the end of this time the emission will decay rather suddenly to a very low value. Inspection may reveal the fact that all of the oxide has disappeared, but frequently the cathode will be found to present much the same appearance as it did at the beginning of its life.

3. *Thermal Properties of Alkaline Earth Oxides*

(a) Measurement of Temperature

The strong temperature dependence of most properties of oxides makes good temperature determinations highly desirable. Since the thermal conductivities of oxides are very low compared with those of metals, methods of thermometry which depend on holding thermocouples or resistance thermometers in contact with an oxide surface result in excessive cooling of the point of contact and thus yield false values of temperature. Hence oxide surface temperatures are usually determined by methods depending on the radiation properties of the surface. Three standard methods are available:

This review is divided into three parts. A complete table of contents is given below. Part I, in this issue, includes the first four sections. Parts II and III will appear in the November and December issues. A complete bibliography will be printed at the end of the series. To assist readers of the first two parts, an abbreviated bibliography for each part is included.

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TABLE I. *Properties of the alkaline earths.*

	BARIUM	STRONTIUM	CALCIUM
Atomic weight	137.36	87.63	40.08
Melting point	1123°K (?)	1030°K (?)	1083°K (?)
Heat of sublimation in kilocalories	44.5 (Van Liempt, 1936)	37.8 (Heat of vaporization, Kelley, 1935)	42.9 (Kelley, 1935)
Vapor pressure: log P (mm Hg) =	$7.83-\frac{9727}{T}$ (Van Liempt, 1936)	$11.98-\frac{8250}{T}-1.27 \log T$ (Kelley, 1935)	$10.78-\frac{9350}{T}-0.66 \log T-1.5\times10^{-4} T$ (Kelley, 1935)
Rate of evaporation log M (g per sq. cm per sec.) = (Calculated from vapor pressure)	$7.67-\frac{9727}{T}-0.5 \log T$	$11.72-\frac{8250}{T}-1.77 \log T$	$10.35-\frac{9350}{T}-1.16 \log T-1.5\times10^{-4} T$
Ionization potential (Electron volts)	5.19	5.67	6.09
Work function (Electron volts)	2.52 (Anderson, 1938, cf. also Cashman & Bassoe, 1939)	2.1 (Becker, 1935)	3.2 (Becker, 1935)

(i) Direct pyrometer measurement of the surface temperature of the coating. For optical pyrometry a large correction must be made for the spectral emissivity of the coating. For total radiation pyrometry the total emissivity must be known.

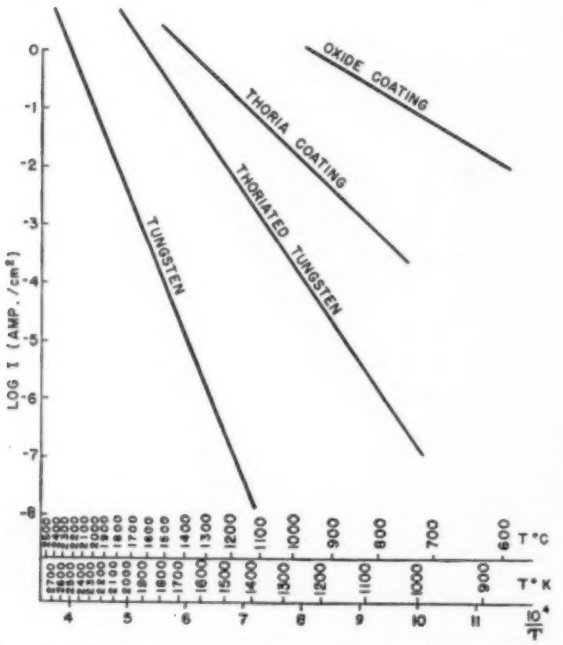

FIG. 1. Electron emission from oxide coating compared with emission from other sources.

(ii) Measurement of power input to the filament. If the total emissivity is known, we may write

$$\text{Power input per sq. cm of surface} = e\sigma T^4, \quad (3.1)$$

where e = total emissivity and σ = Stefan's constant = 5.72×10^{-5} erg/cm²/sec./deg.$^{-4}$. As we shall see below, e is not constant, but is also a function of temperature.

(iii) Deduction of oxide surface temperature from temperature of core metal. If the core metal is partly exposed, its temperature can be measured by a pyrometer. If not, its temperature can be deduced from its electrical resistance. Table III is a summary of brightness temperatures and electrical resistivities as functions of true temperature for several metals commonly used as bases.

The temperature T of the oxide surface can now be calculated by equating the quantities of heat arriving at and leaving the oxide surface:

$$k/d(T'-T) = e\sigma T^4, \quad (3.2)$$

where T' = temperature of core metal, k = thermal conductivity of oxide, d = thickness of oxide, e = total emissivity, and σ = Stefan's constant.

The difference $T'-T$, i.e., the temperature

TABLE II. *Properties of the alkaline earth oxides.*

	BaO	SrO	CaO
Molecular weight	153.36	103.63	56.08
Melting point (Schumacher, 1926)	2196°K	2703°K	2845°K
Heat of sublimation in kilocalories	90 (Claassen & Veenemans, 1933)	117 (See Sec. 3(*d*))	128 (See Sec. 3(*d*))
Vapor pressure log P (mm Hg) =	$8.87-\frac{19{,}700}{T}$ (Claassen & Veenemans, 1933)	$10.00-\frac{25{,}500}{T}$ (See Sec. 3(*d*))	$10.40-\frac{28{,}000}{T}$ (See Sec. 3(*d*))
Rate of evaporation log M (g per sq. cm per sec.) = (Calculated from vapor pressure)	$8.73-\frac{19{,}700}{T}-0.5\log T$	$9.77-\frac{25{,}500}{T}-0.5\log T$	$10.04-\frac{28{,}000}{T}-0.5\log T$
Heat of formation (kilocalories)	133	141	152
Heat of dissociation* of vapor (kilocalories)	147	125 (approx.)	126 (approx.)

* Heat of dissociation of vapor calculated from the thermochemical cycle involving the heat of sublimation of the oxide, the heat of dissociation of the oxide vapor, the heat of sublimation of the metal, the heat of dissociation of O_2 (118 kcal.) and the heat of formation of the oxide.

drop through the oxide coating, may be as much as 100° or even more, so it is by no means safe to assume that the surface temperature of the oxide is the same as that of the core metal.

(b) Emissivity

All three methods mentioned above require a knowledge of one or other type of emissivity. Unpublished measurements made in the laboratory of the General Electric Company indicate that the spectral emissivity for $\lambda=6650$A (used in optical pyrometry) is between 0.5 and 0.7 for thin coatings of (BaSr) oxides on metal bases. This results in corrections of the order of 30° at 1100°K to obtain true temperatures from pyrometer temperatures.

The spectral emissivity e_λ can be deduced from the spectral reflectivity r_λ, since

$$e_\lambda+r_\lambda=1.$$

Prescott and Morrison[18] have described a convenient method for measuring reflectivity. The cathode is supported in an enclosure whose walls are covered with a diffuse reflector (e.g., white velvet). The interior of the enclosure is illuminated by several incandescent lamps. The cathode and that part of the wall immediately behind it are now viewed through an optical

TABLE III. *Melting points, brightness temperatures and resistivities of common core metals.*

	Brightness temperature in degrees Kelvin*					Resistivity ohm-cm $\times 10^6$†		
T°K	W	Mo	Ta	Pt	Ni	W	Mo	Ta
273						5.00	5.14	12.56
300						5.65	5.78	13.85
400						8.06	8.15	
500						10.56		
600						13.23		
700						16.09		
800						19.00		
900						21.94		
1000	966	958	967	950	956	24.93	23.9	44.1
1100	1058	1049	1060	1037	1047	27.94		47.3
1200	1149	1139	1152	1124	1137	30.98	29.5	51.0
1300	1240	1228	1242	1211	1226	34.08		54.8
1400	1330	1316	1332	1296	1315	37.19	35.2	59.0
1500	1420	1403	1421	1381	1403	40.36		62.4
1600	1509	1489	1508	1466		43.55	41.1	65.8
1700	1597	1574	1596	1551		46.78		69.3
1800	1684	1658	1682	1634		50.05	47.0	72.5
1900	1771	1741	1767	1717		53.35		75.8
2000	1857	1824	1852			56.67	53.1	78.9
2100	1943	1905	1933			60.06		82.0
2200	2026	1986	2018			63.48	59.2	85.2
2300	2109	2065	2099			66.91		88.3
2400	2192	2143	2181			70.39	65.5	91.3
2500	2274	2220	2261			73.91		94.4
2600	2356	2297	2341			77.49	71.8	97.4
2700	2437	2373	2421			81.04		100.2
2800	2516	2448	2499			84.70		102.9
2900	2595		2575			88.33		105.6
3000	2673		2652			92.04		108.7
3200	2827		2803			99.54		113.9
3400	2978					107.2		
3600	3165					115.0		
Melting Point °K	3655	2895	3269	2046	1725			

* *Brightness Temperatures:* W: Forsythe and Worthing, Astrophys. J. **61**, 146 (1925); Mo: Worthing, Phys. Rev. **28**, 190 (1926); Ta: Malter and Langmuir, Phys. Rev. **55**, 743 (1939); Pt and Ni: Forsythe, Int. Crit. Tab. Vol. **V**, p. 245.

† *Resistivity:* W: Jones and Langmuir, Gen. Elec. Rev. **30**, 354 (1927); Mo: Worthing, Phys. Rev. **28**, 190 (1926); Ta: Malter and Langmuir, Phys. Rev. **55**, 743 (1939).

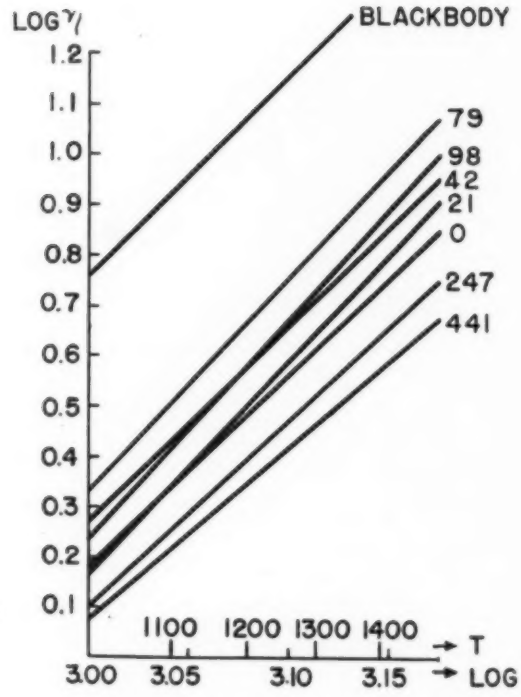

FIG. 2. Logarithmic plot of total radiation η in watts per sq. cm of BaO+SrO (59 molecular percent SrO) on a nickel core as a function of temperature. Thickness in μ indicated at right (Clausing and Ludwig, reference 12).

pyrometer. If S and T are the apparent temperatures of cathode and background, as measured by the pyrometer, the reflection coefficient can be deduced from an application of Wien's formula:

$$\ln r_\lambda = \frac{C_2}{\lambda}\left(\frac{1}{T}-\frac{1}{S}\right),$$

where $C_2 = 1.432$ cm deg. If the cathode is hot, a further term is necessary in this formula to include the fact that the cathode is radiating as well as reflecting light of wave-length λ.

Unfortunately Prescott and Morrison applied this method only to a special type of cathode in which the oxide coating was impregnated with finely divided nickel. Consequently the value of 0.64, which they obtain for the spectral emissivity, is probably not of very wide applicability.

For the total emissivity a certain amount of data is available. Dushman[9] quotes the relation between emissivity and temperature used by the Western Electric Company in connection with their vacuum tubes:

$$e = 0.4 + 2.5\times10^{-4}T(800\text{–}1200°\text{K}).$$

This applies to a coating of a mixture of barium and strontium oxides on a core of 95 percent Pt+5 percent Ni.

Patai and Tomaschek[15] measured total emissivity for cathodes prepared by deposition from a colloidal suspension and obtained $e=0.5$. This figure was deduced from observations of the change of surface temperature of an oxide cathode in a vacuum tube, when gas of known thermal conductivity was introduced.

The most exhaustive study as yet published of the emissivity of (BaSr) oxides is due to Clausing and Ludwig,[12] who studied coatings of various thicknesses of SrO and of BaO+SrO on nickel strips. Their experiments involved heating their whole experimental tube, which was constructed of quartz, to the temperature of the filament, and so avoiding temperature gradients in the oxide coatings. They showed that in general the total emission can be described by an expression of the form

$$\eta = pT^q, \qquad (3.3)$$

where p has values between 10^{-3} and 10^{-8} erg/cm^2/sec./deg.^{4}, and q varies from 3.3 to 4.9, depending on the oxide thickness.

A few of their measurements of η as a function of temperature are given in Fig. 2. If we plot the ratio of total emission to the blackbody emission for the same temperature (i.e., total emissivity) against thickness of coating, their results give a curve of the form shown in Fig. 3. From the zero thickness value characteristic of clean nickel, the emissivity increases to a maximum of about 0.38 for thicknesses of about 75μ (about 10 mg of carbonate coating per sq. cm) and then decreases until at thicknesses of 300μ or more the emissivity has reached a more or less constant value of about 0.2.

These authors obtained measurements of spectral emissivity ($\lambda=0.665\mu$) which indicate for thin coatings (around 30μ) a value of 0.5–0.6. With increasing thickness the spectral emissivity drops off sharply to values as low as 0.1 for coatings thicker than 150μ.

From the collected experimental data we are probably safe in saying for the ordinary oxide-coated cathode, whose coating is about 4×10^{-2} cm thick (about 4 mg of oxide per sq. cm),

Spectral emissivity ($\lambda=0.665\mu$) = about 0.5

Total emissivity = about 0.3.

(c) Thermal Conductivity

If the emissivity of an oxide coating is known, determinations of power input and temperature of base metal suffice for a calculation of the thermal conductivity of the coating. The surface temperature of the oxide can be deduced from Eq. (3.1) above. By substituting this value and the known temperature T' of the base metal, in Eq. (3.2) we can calculate the thermal conductivity k.

Patai and Tomaschek[15] quote values of 0.00012 to 0.0013 cal. grad.$^{-1}$ cm^{-2} sec.$^{-1}$ for the thermal conductivity of rather dense coatings deposited from a colloidal suspension. Clausing and Ludwig[12] give the single value 0.00034 cal. grad.$^{-1}$ cm^{-2} sec.$^{-1}$. No data are available regarding the variation of this quantity with temperature.

(d) Rates of Evaporation and Vapor Pressures

Since the life of a cathode frequently is terminated by the evaporation of its oxide coating, it is not surprising that a number of investigations (Arnold,[3] Thompson and Armstrong,[7] Reerink (cf. Zwikker[7]) and Claassen and Veenemans[12]), have been devoted to determining the rates of evaporation of the alkaline earth oxides. The best measurements have been made on pure BaO, since its vapor pressure is higher by a factor of about 1000 than those of SrO and CaO. For some time the observations of Claassen and Veenemans have been accepted as the most accurate for BaO. Recently, however, their results have been challenged by Herrmann.[16] The two sets of observations agree around 1400°K; above and below this point they diverge, Herrmann's curve corresponding to a heat of sublimation of 119 kcal., while Claassen and Veenemans measure only 90 kcal.

In measurements of this type the usual stumbling block is the temperature determination. Claassen and Veenemans evaporated the oxide from the inside of a platinum cylinder heated by high frequency, onto a cooled quartz surface. The temperature of the outside of the platinum cylinder was obtained by pyrometer measurements and was taken to be the temperature of the oxide surface. If an appreciable thermal gradient existed through the oxide coating, the results might be seriously in error. Herrmann attempted to avoid this difficulty by pyrometering a spot of Cr_2O_3 deposited on the surface of the oxide. The emissivity of Cr_2O_3 is known to be about 0.8 and remains constant with temperature. This method is open to further objection (cf. Blewett, Liebhafsky, and Hennelly[18]), since the Cr_2O_3 spot cools itself below the oxide temperature due to its higher emissivity and so fails to give a true measure of the oxide temperature.

Since these considerations do not permit an unequivocal decision between these conflicting data, the method of Knudsen was applied by Blewett, Liebhafsky, and Hennelly.[18] Within the experimental errors the results check the values quoted by Claassen and Veenemans. The rate of evaporation of BaO is given by

$$\log m\sqrt{T} = -19400/T + 8.48$$
(m in g per cm^2 per sec.).

From the kinetic theory relation $p = m(2\pi RT/M)^{\frac{1}{2}}$

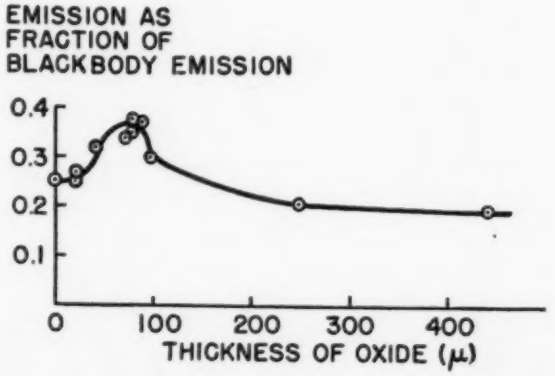

Fig. 3. Total emissivity of BaO+SrO as a function of thickness of coating (T=1100°K) (Clausing and Ludwig, reference 12).

the vapor pressure of BaO is found to be given by

$$\log p = -19400/T + 8.63 \text{ mm Hg.}$$

Measurements on the rates of evaporation of SrO and CaO were made by Claassen and Veenemans,[12] using the same apparatus that was used for BaO. The points which they obtained scatter rather badly, since the rates of evaporation are so low as to be barely measurable. However, a reasonable value for the heat of vaporiza-

tion for these substances can be deduced from a single measurement of vapor pressure by the following thermodynamic argument:

The free energy change when a solid evaporates is given by

$$\Delta F = -RT \ln P = \Delta H - T\Delta S,$$

where P is the vapor pressure, ΔH is the heat of sublimation, and ΔS is the change in entropy. With the assistance of the third law of thermodynamics, the entropy data collected by Kelley (Bull. 394),[14] and the assumption that the difference in specific heat between solid and vapor is $3R$ (cf. Mayer and Wintner[17]), it is possible to evaluate ΔS and hence ΔH. We obtain

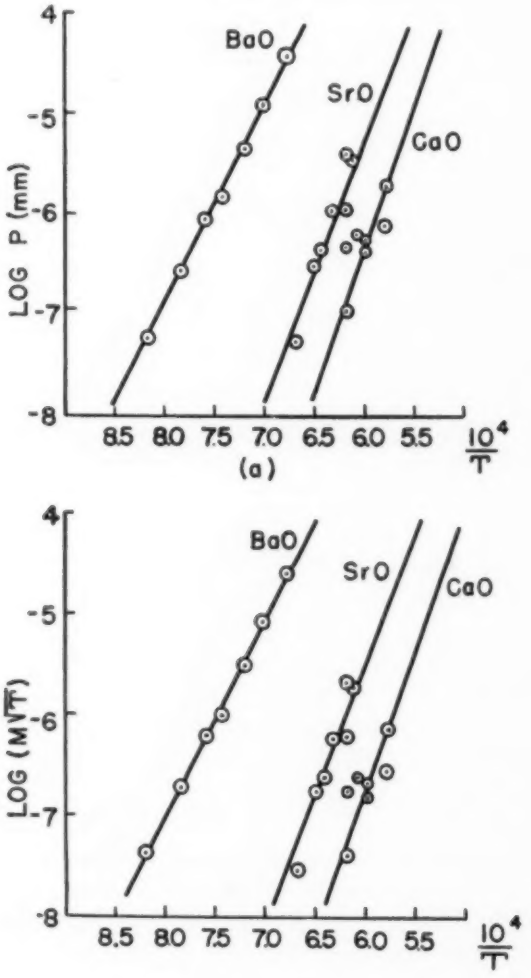

FIG. 4. (*a*) Vapor pressures of the alkaline earth oxides. (*b*) Rates of evaporation of the alkaline earth oxides (Claassen and Veenemans, reference 12).

for SrO, $\Delta H = 117$ kcal.
for CaO, $\Delta H = 128$ kcal.

with possible errors of about eight percent.

In Fig. 4 we have drawn lines consistent with this argument through Claassen and Veenemans' experimental points, for evaporation of SrO and CaO, instead of the lines which these authors chose as the best fit for their experimental points.

BaO evaporates preferentially from a mixture of BaO and SrO (Claassen and Veenemans[12]) to leave a surface layer of SrO alone. This has been adequately demonstrated by the electron microscope studies of Gaertner[14] and of Darbyshire.[17] The properties of mixtures of these oxides are considered in Section 6 below.

4. *Electrical Properties of Alkaline Earth Oxides*

(a) Electrical Conductivity; Theory of Conductivity and Thermionic Emission

Some idea of the confusion which exists with respect to the electrical conductivity can be obtained from Fig. 5, which represents measurements to be found in the literature for the conductivity of BaO or of mixtures of BaO and SrO. Order emerges, however, when a few factors are considered. Horton's early measurements[2] and those of Spanner[4] were made in air and so probably apply to a mixture of BaO with other barium compounds. The measurements of Becker,[10] Reimann and his collaborators,[9, 10] and Clausing (cf. de Boer, reference 14, p. 361) were made on oxide coatings on filaments in the presence of steep thermal gradients. Thermal gradients, particularly at lower temperatures, result in marked polarization effects which can obscure the values obtained. Frequently apparent deviations from Ohm's law are observed, so that the conductivity appears to depend on the applied voltage. Kroczek and Lübcke[9] measured conductivities of oxide coatings by an ingenious method involving a balance between the heating effect of the conduction current through the oxide and the cooling effect of the electron emission. Their experimental values for conductivity scatter rather badly, probably because their various specimens were prepared in different ways, so that the line bearing their names in Fig. 4 is a very qualitative representation of their results. Blewett's measurements were made

on unactivated BaO in bulk, rather loosely packed. The measurements of Meyer and Schmidt[11] were made on a thin layer of BaO pressed between two blocks of nickel which were maintained at the same temperature. Polarization difficulties were avoided by the use of high frequency alternating current, so these measurements are probably the best available. It must be emphasized, however, that a specific conductivity cannot be assigned to a substance which, like BaO, has not been prepared in the form of large single crystals. Measured conductivities have meaning only when the particle size of the powder, its state of compression, and the amounts of impurity are completely specified.

Kroczek and Lübcke[9] have observed a decrease in the conductivity of barium oxide as the particle size is increased, indicating that surface conduction makes an important contribution to the total conductivity.

Little can be said regarding the electrical conductivities of SrO and CaO. Measurements on these oxides (Horton[2] and Spanner[4]) are open to objection, since they were made in air, but they indicate a conductivity for SrO an order or so lower than that of BaO, and a value lower still for CaO.

The measurements of Meyer and Schmidt[11] show that activated barium oxide has a considerably higher conductivity than it does in the unactivated state. When barium oxide is activated, it is believed that an excess of free barium is present, so that the increased conductivity is analogous with that observed in other semiconductors in the presence of a stoichiometric excess of their electropositive components.

The question of the nature of the conductivity has been discussed by many authors. The rate of activation of an oxide cathode is usually accelerated if electron current is drawn from it, suggesting that an electrolytic current passes through the oxide and liberates free (BaSrCa) at the surface of the core metal. Harris (cf. Becker[8]) has detected barium and strontium which actually penetrated the core of an oxide cathode from which a heavy current had been drawn. When the cathode was merely glowed without drawing current, no barium or strontium was found in the core. If, on the other hand, the direction of the current through the oxide is reversed by supplying electrons to the cathode from an external source, electrolysis should result in deposition of a layer of free barium on the surface of the oxide. Essentially the same changes in activity should be observed as when barium is deposited from an external source. Becker[8] and Becker and Sears[10] have shown that this is indeed the case. They were further successful in showing that the rate of appearance of barium at the surface is proportional to the quantity of charge sent through the oxide. This was demonstrated by plotting activity of cathode against quantity of charge passed. For widely different currents the observed points lay on the same curve. From these measurements Becker concludes that about 0.4 percent of the current at 800°K is carried by barium ions. This figure involves the assumption that activation is due to a monatomic layer of free barium on the oxide surface and includes a very rough estimate of the total surface area of the oxide. Berdennikowa[11] made chemical measurements of the total amount of barium set free (see Section 8(a) for the method employed) and agrees that Faraday's first law is obeyed, the ratio of barium ion current to total current being about 5×10^{-5} at 1273°K.

During the activation of an oxide cathode a good deal of oxygen also is liberated. This evolution of oxygen has been established by the experiments of Detels,[6] who identified the gas by its spectrum in a Geissler tube, and by Becker,[8] who detected it by its effect on the electron emission from an adjacent tungsten filament. Isensee[16] has made a chemical determination of the rate of evolution of oxygen during voltage activation of an oxide coating on a platinum filament. From his measurements he concludes that at the beginning of activation of a cathode coated with a mixture of alkaline earth oxides about 0.2 percent of the current through the oxide is carried by oxygen ions. This fraction drops to less than 0.001 percent when the cathode is fully activated. With a pure barium oxide coating the oxygen ion current maintained a steady value of about 0.05 percent of the total current. Pure strontium oxide gave no detectable electrolytic evolution of oxygen. Becker also observed that the rate of oxygen evolution from a mixed oxide coating was considerably less when

the filament became fully activated. He noted also that oxygen evolution ceased when the emission current became space-charge limited—i.e., when no electric field existed at the surface of the oxide. Since the potential of the detecting filament had no apparent effect on the rate of arrival of oxygen, he concluded that the oxygen escapes as neutral atoms or molecules. A small fraction of the oxygen, however, has been shown by the mass spectrograph to escape in the form of negative ions (cf. Section 4(c)). The peculiar dependence of oxygen evolution upon surface electric field indicates that the process is more complex than a simple electrolysis. It is rather difficult to imagine a mechanism by which the oxygen ions arriving at the oxide surface could get rid of their surplus electrons to evaporate as neutral atoms, since oxygen has a large electron affinity. If the atoms can combine to form molecules, however, the required energy would be available in the molecular heat of formation. Again, however, it seems improbable that two negative ions could approach each other sufficiently closely to effect this combination. Possibly the presence of an electric field is necessary to promote this process.

From these results it is clear that the electrolytic part of the conductivity, although important in the activation process, is comparatively small, so that for most purposes the oxide may be considered to be an electronic semi-conductor. In semi-conducting materials like barium oxide the number of electrons available for conduction or electron emission is small compared with the total number of atoms. This simplifies the theoretical discussion of these properties, since the electron energy distribution given by Fermi statistics reduces effectively to the classical distribution.

The alkaline earth oxides fall into the classification of "impurity semiconductors," the impurity in this case taking the form of scattered atoms of the free alkaline earth metal. These free atoms provide localized energy levels in the forbidden region between the filled electron band and the conduction levels. As can be seen from Fig. 6, the transfer of an electron from an impurity level to the conduction band requires an energy Q_1 which is less than the energy Q_0 required to extract an electron from the filled band. In the temperature range with which we

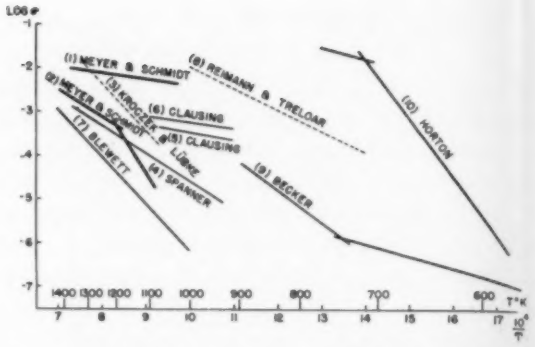

FIG. 5. Electrical conductivity of BaO and BaO+SrO. (1) Meyer and Schmidt, reference 11. Activated BaO. (2) Meyer and Schmidt, reference 11. Unactivated BaO. (3) Kroczek and Lübcke, reference 9. Conductivity of oxide coating on cathode. Line given here is drawn through a number of widely scattered points obtained from measurements on different cathodes. (4) Spanner, reference 4. Conductivity in air of BaO in bulk. (5) Clausing (de Boer, reference 14, p. 361). Activated BaO on cathode. The temperature range over which these measurements and those of curve (6) were made is not given in the reference. (6) Clausing (de Boer, reference 14, p. 361). Conductivity of BaO on cathode after running for 1000 hours. (7) Blewett (unpublished). Conductivity in vacuum of BaO in bulk, rather loosely packed. (8) Reimann and Treloar, reference 10. Conductivity of BaO+SrO coating on filament. Thickness of oxide is not given in the reference, so a value of 0.5 mm was assumed in plotting this line. (9) Becker and Sears (cf. Becker, reference 10). Conductivity of BaO+SrO coating on cathode. (10) Horton, reference 2. Conductivity in air of BaO in bulk.

are concerned it is to be expected that the supply of electrons from impurity levels will so far outweigh the supply from the filled band that only the former need be considered.

We shall now proceed to evaluate the number n_c of electrons in the conduction band at temperature T°K. The number of electrons n_1 transferred in unit time to the conduction band from impurity levels will be proportional to the number of impurity atoms N_1 and to a Boltzmann factor. Thus:

$$n_1 \sim N_1 e^{-Q_1/kT}.$$

The number of electrons n_2 returning in unit time from the conduction band will be proportional to the number of electrons n_c in the conduction band and to the number of vacant impurity levels, which also is n_c:

$$n_2 \sim n_c^2.$$

In the equilibrium state $n_1 = n_2$. Hence

$$n_c^2 \sim N_1 e^{-Q_1/kT}$$

and

$$n_c \sim N_1^{\frac{1}{2}} e^{-Q_1/2kT}. \qquad (4.1)$$

The thermionic emission from the semiconductor will now be composed of those electrons in the conduction band which have sufficient energy to cross the potential barrier at the surface of the crystal. Let the height of this barrier be φ (see Fig. 6)—i.e., φ is the difference in potential energy between an electron in space just outside the crystal and an electron in the conduction band. The number of electrons whose energy is greater than φ will be proportional to the number of electrons in the conduction band and to a Boltzmann factor, so that the rate of emission of electrons will be proportional to

$$n_c e^{-e\varphi/kT};$$

i.e., to

$$N_1^{\frac{1}{2}} e^{-(1/kT)(\varphi+Q_1/2)}.$$

The complete expression for the thermionic emission includes a temperature factor and takes the form (neglecting statistical weight factors of the order of unity (cf. Schottky[14] or Fowler, reference 15, Section 11.62)):

$$I = A' D N_1^{\frac{1}{2}} T^{5/4} e^{-1/kT(\varphi+Q_1/2)}, \qquad (4.2)$$

where

$$A' = e\left\{\frac{(2\pi)^{\frac{1}{2}} k^{5/2} m^{\frac{1}{2}}}{h^3}\right\}^{\frac{1}{2}},$$

$=$ about 10^{-6}, if I is to be expressed in amperes per sq.cm,

$D =$ mean transmission coefficient of oxide surface.

We are now in a position to interpret some of the experimental data. The electrical conductivity σ of a material is known from the elementary theory of conduction to be proportional to $ln_c/\sqrt{T}$ where l is the electronic mean free path. Compared with n_c, which is strongly temperature dependent (Eq. (4.1)), the quantity $l/\sqrt{T}$ is effectively a constant, and a semilog plot of σ against reciprocal temperature will be a straight line whose slope is proportional to Q_1. The data plotted in Fig. 5 indicate that Q_1 decreases from a value of about 5 volts, characteristic of the stoichiometric (unactivated) compound, to about 1.6 volts when the oxide is fully activated by the introduction of free barium "impurities." Given Q_1, we can deduce φ from the measured values of the thermionic work function $(\varphi+Q_1/2)$ (cf. Section 10(a) below). The best values for $(\varphi+Q_1/2)$ are around 1.2 volts for barium oxide, so that φ is probably about 0.4 volts (cf. Reimann and Treloar,[10] who measured the electron emission and conductivity of the same oxide specimen).

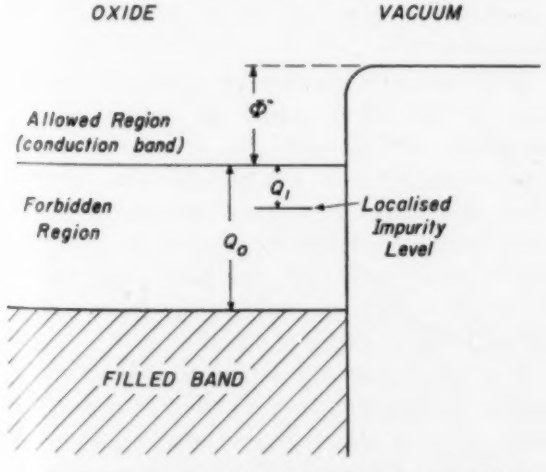

FIG. 6. Energy level distribution for an impurity semiconductor.

The expressions (4.1) and (4.2) above show that both conductivity and thermionic emission are proportional to the square root of the number of impurity atoms. This indicates that during the activation process the conductivity for a fixed temperature should be linearly proportional to the electron emission. A linear relation of this type has been observed by Albricht.[10]

The behavior of the transmission coefficient D will be discussed in Section 10(a).

(b) THERMOELECTRIC EFFECT

The theory of the thermoelectric effect in semiconductors is given by Fowler, reference 15, Section 11.78. The expression which he deduces for the thermal e.m.f. of a circuit consisting of a semi-conductor and a normal metal with junctions at temperature T and $T+\Delta T$ can be reduced, if ΔT is small compared with T, to the approximate form

$$E = 2\times10^{-4}\Delta T(7+\log_{10} lT/\sigma) \text{ volts}, \qquad (4.3)$$

where σ is the conductivity and l is the electron mean free path.

Experimental studies of the thermoelectric effect in various semiconductors have been carried out by Mönch[15] and other workers in the University of Erlangen (cf. Fritsch[14] and Bauer[15]). These workers deduce the electron mean free path from simultaneous measurements of the

conductivity and the Hall coefficient R, using the theoretical relation

$$R\sigma = 2.84\times10^{9}l/\sqrt{T}. \qquad (4.4)$$

l is usually found to be of the order of 10^{-7} cm, so that for a temperature of 1000°K

$$E = 2\times10^{-4}\Delta T(3-\log\sigma) \text{ volts}. \qquad (4.5)$$

Simultaneous measurements of conductivity and thermal e.m.f. of specimens of barium oxide have been made by the author in the laboratory of the General Electric Company. The results obtained are consistent with Eq. (4.5). The values for the thermal e.m.f. vary between 9×10^{-4} and 3×10^{-3} volt per degree depending on the state of preparation and activation of the oxide. Becker and Sears[10] observed a thermal e.m.f. of 0.05 volt for a temperature difference which they estimate to be about 20°. This represents a thermal e.m.f. of 2.5×10^{-3} volt per degree in agreement with the above measurements.

(c) Ion Emission

If (BaSrCa) oxides are heated to temperatures of the order of 1800°K, currents of positive ions of the order of 10^{-7} amp. per cm^2 can be drawn to a negative collector. Mass-spectrograph analyses (Blewett and Jones[15]) show that the ion current, aside from sodium and potassium ion impurities, consists of singly charged ions of (BaSrCa). This, however, is not necessarily evidence for electrolytic conductivity, since the alkaline earth may be set free by other mechanisms and subsequently ionized.

The alkaline earth oxides are found also to emit negative ions (Barton,[5] Blewett and Jones,[15] Bachman and Carnahan,[17] and Broadway and Pearce[18]). These ions are responsible for the unpleasant "black spot" phenomenon observed in cathode-ray tubes employing magnetic deflection. The ion beam, which is relatively unaffected by the deflecting fields suitable for electrons, falls continually on a spot in the center of the screen, and after a short time the bombarded region loses its fluorescent properties. Analysis of the ion beam by mass-spectrograph techniques shows that a large number of components are present. Atomic and molecular oxygen ions are observed and may be due to the oxide. In addition, however, extraneous ions appear whose mass numbers correspond to hydrogen, carbon, and halogen impurities, as well as a number of others which cannot be so readily identified. Possibly some decrease in this troublesome effect could be achieved by the use of extremely pure oxide coated on a carefully purified base metal with the assistance of a binder which can be trusted to evaporate completely during the initial outgassing procedure.

(d) Behavior of Tungsten Filaments in BaO Vapor

The electron emission of a tungsten filament maintained at a temperature of 1000°K or less in a stream of BaO vapor was studied by Davisson and Pidgeon.[3] These authors observed that, as BaO condenses on the filament, its electron emission increases by several powers of ten to reach a maximum when something of the order of a monolayer of BaO has been deposited. The emission then falls off, only to increase again and pass through a second maximum, after which it decays gradually to a more or less constant value. This peculiar behavior has also been observed in this laboratory. Unpublished experiments, performed in the laboratory of the General Electric Company by G. L. Tawney, suggest that the double maximum will merge into a single one if the vacuum conditions are sufficiently good. His experiments were carried out in a well-gettered tube immersed in liquid nitrogen. It is probable that the double maximum is of the same nature as that observed by Taylor and Langmuir[12] when caesium is deposited on imperfectly cleaned tungsten.

If the tungsten filament is raised to a temperature of 2000°K or more, the impinging BaO molecules will be observed to re-evaporate in the form of positive ions. For high filament temperatures about five percent of the incident BaO molecules are converted to positive ions. It is highly improbable that the BaO molecules are ionized as such, since their ionization potential is probably of the order of 10 volts.

A more probable reaction is that involving capture by the tungsten filament of the oxygen atom and an electron, and evaporation of a positive ion of barium. The energy required for this process can be deduced from a thermo-

chemical cycle involving the heat of evaporation of oxygen from tungsten (162 kcal., Langmuir and Villars[10]), the work function of tungsten (104 kcal.), the heat of dissociation of barium oxide (147 kcal., Table II), and the ionization potential of barium (119 kcal.). The net energy required is

$$-162-104+147+119=0\pm 10 \text{ kcal.}$$

(The greater part of the probable error enters through the heat of dissociation of barium oxide.) Since the necessary energy is so small, the reaction is a probable one. At the temperature at which maximum ionization takes place the electron emission of the filament is less than that of clean tungsten, as would be the case in the presence of a layer of adsorbed oxygen.

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The Filtration of Sound. II.

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In this second article on the Filtration of Sound, Professor Lindsay describes particularly filtration in solid media. It contains also a survey of recent developments.

1. Acoustic Filtration in Solids. Loaded Rods

THE filtration structures considered in Part I[1] employ air or some other gas as the propagation medium. However, the general theory is of such a character that it should apply to elastic waves in any material medium containing periodic discontinuities or nonhomogeneities of structure. We shall here confine our attention to solid structures.

A simple type of solid filter is provided by a solid rod loaded at equal intervals with heavy masses.[2] This is illustrated in Fig. 1 by an infinite rod with heavy collars attached to it. Let the mean equilibrium density of the rod material be ρ_0, its area of cross section S and the mass of each load m. The distance between successive loads is $2l$. We shall try to keep the description of the transmission of linear compressional harmonic waves through the structure as analogous as possible to that of the air filter structures treated in Part I. With this in mind we shall use the volume displacement velocity or volume current $\dot{X}=S\dot{\xi}$ as before, and denote the excess *compressive* stress (the negative of the excess dilatational stress) by p. Hooke's law then takes the form

$$p\Big/\frac{\partial\xi}{\partial x}=-Y=-\rho_0 c^2, \tag{1}$$

where Y is Young's modulus and c is the velocity of a linear compressional wave through the rod. We can now express the excess compressive stress and the volume current at the successive mid-section points $j+1$ and j in precisely the same fashion as for the air structure. Thus (cf. Eqs. (1) and (2) of Part I)

$$p_{j+1}=A'p_j-iB'Z\dot{X}_j, \tag{2}$$

$$\dot{X}_{j+1}=A'\dot{X}_j-iC'/Z\cdot p_j, \tag{3}$$

with $Z=\rho_0 c/S$ as usual. The parameters A', B', and C' are now given by

$$\begin{aligned} A'&=\cos 2kl-\omega m/2ZS^2\cdot\sin 2kl\\ B'&=\sin 2kl+\omega m/ZS^2\cdot\cos^2 kl\\ C'&=\sin 2kl-\omega m/ZS^2\cdot\sin^2 kl. \end{aligned} \tag{4}$$

It is worth while to pause a moment to consider the boundary conditions leading to (2), (3) and (4). Consulting Fig. 2, we denote quantities in the rod just to the left of the (arbitrarily chosen) first load by the subscript 12, while corresponding quantities just to the right are denoted by 21. The continuity of displacement, assuming that the rod moves as a whole, is then expressed by

$$\dot{\xi}_{12}=\dot{\xi}_{21}=\dot{\xi}_L, \tag{5}$$

where $\dot{\xi}_L$ is the displacement velocity of the load. The stress boundary condition is here simply the equation of motion of the load. Thus

$$S(p_{12}-p_{21})=m\ddot{\xi}_L=i\omega m\dot{\xi}_{12}=i\omega m\dot{\xi}_{21}. \tag{6}$$

The application of (5) and (6) to the transmission of compressional harmonic waves through the rod leads directly to (2), (3) and (4) by the usual analysis.

As before (Part I, Eq. (4)) we see that

$$A'^2+B'C'=1 \tag{7}$$

and therefore let

$$\begin{aligned} A'&=\cos W,\quad B'=(B'/C')^{\frac{1}{2}}\sin W,\\ C'&=(C'/B')^{\frac{1}{2}}\sin W. \end{aligned} \tag{8}$$

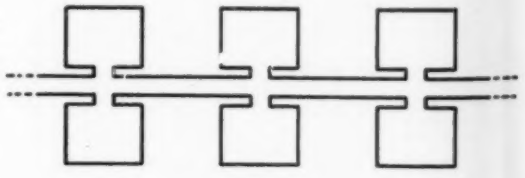

FIG. 1.

The filtration characteristics of the *infinite* structure can now be readily recognized from the analysis of Section 4 of Part I. We get indeed a series of alternating transmission and attenuation bands given by the behavior of cos W. Since cos $W=1$ for $\omega=0$ and then at first decreases as ω increases, the structure is a low-pass compressional wave filter. Indeed the plot of cos W for a special case of a brass rod with $S=0.32$ cm², $2l=25.08$ cm, $m=875$ grams ($c=3.5\times10^5$ cm/sec., $\rho_0=8.5$ grams/cm²) is presented in Fig. 3 and indicates the low-pass nature of the structure.

There is, to be sure, a difference between the characteristics of the low-pass solid filter and the low-pass air filter immediately recognizable from the cos W curves (cf. Fig. 3 of the present paper and Fig. 9 of Part I). The solid filter presents an alternation of pass and attenuation bands, in which after the first pass band the succeeding ones become successively narrower. In fact, from the expression for cos W, we see that transmission can be expected to result at high frequencies only in the neighborhood of the resonance frequencies of the main rod

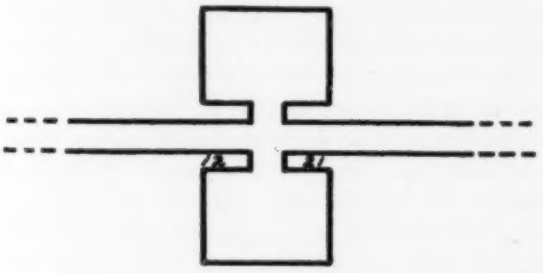

FIG. 2.

section between loads, i.e., for sin $2kl=0$. The fundamental for the illustration cited comes at about 7000 cycles just where the second transmission band is indicated in Fig. 3. We expect another transmission band at 14,000 cycles, etc. As the frequency increases the transmission spectrum becomes, so to speak, more and more nearly a *line* spectrum. In the air filter it is the attenuation bands which form a spectrum of lines growing progressively narrow as the frequency increases.

The filter cited has been studied experimentally by Olson.[3] He found the cut-off at the end of the first transmission band to lie between

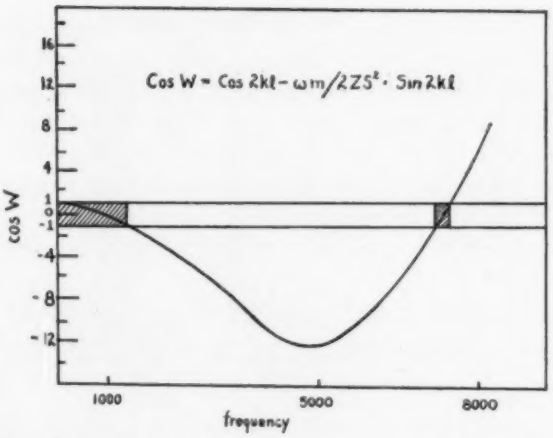

FIG. 3. Transmission characteristic curve for a solid filter.

1200 and 1300 cycles. That computed theoretically from cos $W=-1$ gives 1225 cycles in good agreement. Olson presumably was unable to detect the higher transmission bands.

The analysis of Section 6 of Part I can be at once applied to finite solid filters. For a structure of the loaded rod type considered in the present section inserted in an infinite line consisting of a solid rod of the same material and area of cross section, the theory gives a power transmission ratio

$$P_r=\frac{1}{1+\omega^2m^2/Z^2S^4\cdot\sin^2 nW/4\sin^2 W}. \quad (9)$$

In the first transmission band or region as W goes from 0 to π, the $\sin^2 nW$ and $\sin^2 W$ will be in the interval from 0 to 1 and P_r will be appreciable. In fact we note that at the $n-1$ frequencies corresponding to $W=j\pi/n$ where j goes from 1 to $n-1$, $\sin^2 nW/4\sin^2 W=0$ and we get peaks with $P_r=1$. When we get to $W=\pi$ or cos $W=-1$, increase in frequency yields hyperbolic functions and P_r becomes negligible until once more $|\cos W|\leqq1$ when the behavior in the first transmission band repeats itself over a much smaller frequency range. This is illustrated by the case of a loaded solid rod filter recently studied.[4] Here the material used was steel with $\rho=7.83$ grams/cm³, $2l=20.32$ cm, $S=0.32$ cm², $m=792$ grams, $c=5.14\times10^5$ cm/sec., with $n=6$ (i.e., 6 sections). The approximate plot of P_r out to frequency 13,000 follows. (Fig. 4.) The cut-off frequency computed from cos $W=-1$ comes out to be 2100 cycles, in good

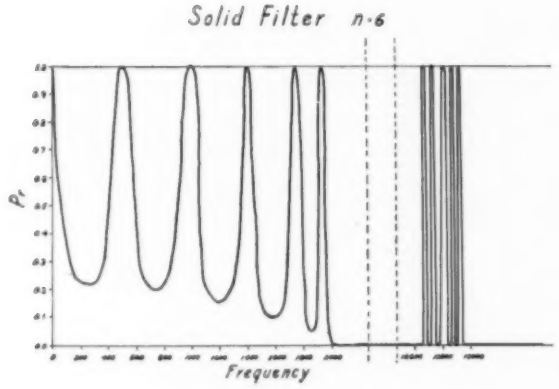

FIG. 4.

agreement with the P_r curve. The experimental cut-off is about 2000 cycles. The second transmission band comes in at about 12,600 cycles but extends to only approximately 13,000 cycles, as the figure clearly shows. Due to the difficulties of the experimental P_r measurement for an equivalent infinite line, the attempt to get P_r over the transmission band was abandoned and P_r' (cf. Sections 7 and 8 of Part I) was measured instead with the filter terminated by a rod 39.75 cm long (with end free). Formula (32) of Part I is appropriate in this case with $S_i = S'$, $Z' = Z$, $C = C'$ (from (4) of the present paper and the cos nW etc. taken also from (4)). Fig. 5 indicates the agreement between theory and experiment. The paper referred to should be consulted for details as to the experimental method. The six infinite resonance peaks in the theoretical curve will be noted. These are to be expected as already mentioned in Section 8 of Part I. In the figure they are cut off at $(P_r')^{\frac{1}{2}} = 6$. Only five experimental peaks could be detected, possibly because the last one is too narrow and hard to bring in. The experimental measurements agree with the theoretical prediction that the attenuation band extends from 2100 to 12,600 cycles. In general the agreement between theory and experiment is good enough to encourage one in designing a low-pass finite solid filter to have preassigned properties. As has already been emphasized in the case of air filters, though the termination has an important influence on the structure of the transmission band, as a general rule it does not greatly influence the cut-off frequency.

2. *Solid Filters. High-Pass Compressional Wave Filters*

In the search for possible high-pass solid filters for compressional waves we are led to the somewhat idealized situation of a horizontal solid rod to which are attached at equal intervals flexible wires with loads of mass m attached to their ends or alternatively solid rods[5] (cf. Fig. 6(a) and (b)). It is assumed that the compressional waves in the main line set up primarily transverse flexural waves in the wires or side rods. The resulting cos W expressions indicate that for *finite* m in the case of the wires and for *free* ends in the case of the rods, the structure acts like a low-pass filter. On the other hand if in case (a) the mass $m \to \infty$ or the ends of the rods in case (b) are rigidly clamped (i.e., attached effectively to ∞ mass) the structure can behave as a high-pass filter if the dimensions are suitably chosen. An analogy may be noted with the case of the air filter. Thus the case of finite m (a) and free end rods (b) compares with the air structure with Helmholtz resonators (i.e., *finite* amounts of air in the branches) leading to low-pass filtration, while the case of infinite m compares with the orifice branch in the air filter (i.e., effectively infinite amount of air in the branch).

Unfortunately these filters in the high-pass arrangement do not appear to be very practical. In particular to secure high-pass filtration if both main and side rods are composed of the same material it is essential that the length of the main line section be very large compared with the length of the side rod. It is also advisable to make the side rods of material of greater Young's modulus than the material of the main rod.

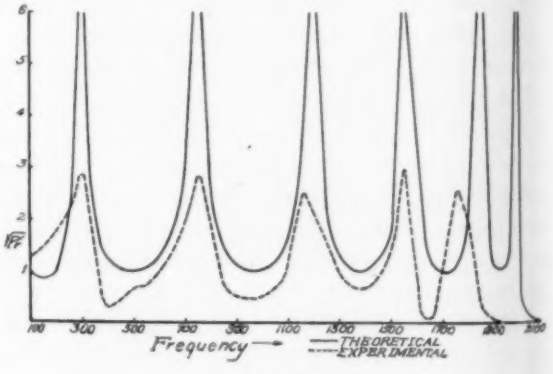

FIG. 5.

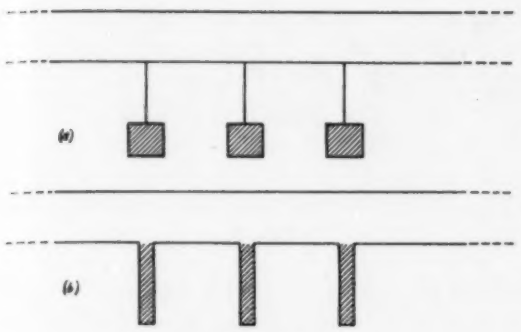

FIG. 6.

Moreover the practical difficulty of securing absolutely rigid clamping of the termini of the side rods must not be overlooked.

Another type of high-pass compressional wave solid filter which may prove more practical than that just described is illustrated schematically in Fig. 7. LL' is the solid rod forming the main line of the structure.[6] This is loaded at equal intervals (length $2l$) with flexible circular membranes M, M', etc., perpendicular to the axis of the rod. These membranes are further attached at their outer edges to the inside of a rigidly mounted hollow circular cylinder CC'. The membranes are stretched with uniform surface tension between rod and cylinder. Longitudinal compressional harmonic waves in the rod set up in each membrane symmetrical harmonic transverse waves whose reaction on the longitudinal waves at the junctions provides the possibility of filtration. Since the membranes are rigidly fastened at their peripheries, we should expect the structure to act as a high-pass filter and analysis indeed confirms this view. Several factors enter into the limiting frequency of the first attenuation band and the problem is somewhat complicated. However we can say that, other things remaining the same, increase in the section length $2l$ decreases the limiting frequency. The latter also appears to vary inversely with the acoustic resistance of the rod material and directly with the tension of the membrane. A concrete example will give an idea of what may be expected. With a main line of hard rubber, 0.5 cm in diameter ($\rho_0=1.3$ grams/cm^3 and $c=1.3\times10^4$ cm/sec.) and flexible rubber membranes 4 cm in diameter with surface density 0.017 gram/cm^2 stretched with tension 7×10^5 dynes/cm, so that the transverse wave velocity is 6.4×10^3 cm/sec., the first attenuation band extends from 0 to 320 cycles for a section length of 2 cm. The succeeding transmission band extends from 320 cycles to 3500 cycles.

Undoubtedly stiff diaphragms may replace the flexible membranes without modifying in essential detail the filtration characteristics of the structure. The analysis of course becomes more complicated.

So far as the author is aware the structures suggested in this section have not been worked with experimentally.

3. *Filtration of Torsional Waves in Solid Rods*

Referring again to the type of structure illustrated in Fig. 1, suppose that harmonic *torsional* waves travel along the main rod, with the twist perpendicular to the axis of the rod. The loads tend to twist as a whole and the structure acts as a low-pass torsional wave filter. The analysis is similar to that in Section 1 with, however, the angular velocity of twist replacing longitudinal particle velocity and the average torque per unit area taking the place of excess compressive stress. The cos W expression is particularly simple, taking the form[5]

$$\cos W=\cos 2kl-\omega I/2ZS\cdot\sin 2kl, \qquad (10)$$

where the notation agrees with that in Section 1, except for I, which is here the moment of inertia of the branch load about the axis of the main rod, and Z, which now becomes the mechanical impedance for a progressive torsional wave, *viz.*

$$Z=S\rho_0 c/2\pi. \qquad (11)$$

Here c is the torsional wave velocity. Examination of (10) discloses the low-pass nature of the filter structure.

A structure of this type has recently been tested experimentally in the physical laboratory at Brown University.[7] The main line was brass (density 8.46 grams/cm^3) 0.475 cm in radius.

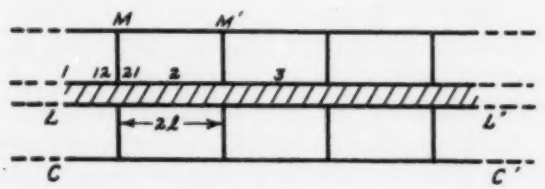

FIG. 7.

The load had a moment of inertia of 869 gram cm². The section length ($2l$) was 19.90 cm. The cut-off frequency computed from $\cos W = -1$ (Eq. (10)) comes out to be 401 cycles. The measured transmission practically vanished at $\nu = 400$ cycles in good general agreement. Actually the structure used was in six sections and was terminated by a cylindrical brass rod of the same diameter as the main line 20 cm long. The power transmission ratio P_r' was calculated by the method of Section 8 in Part I. The comparison between the computed values and the experimentally determined structure of the transmission band is fairly good, though some trouble was caused by the presence of flexural waves. An extended discussion will be published elsewhere.

Another rather interesting variety of torsional wave filter is the structure schematically indicated in Fig. 7 and already discussed in the preceding section as a high-pass compressional wave filter.[8] If harmonic torsional waves travel along the main rod LL', they produce in the loading membranes a radial torsional wave whose interaction with the main line torsional wave gives the structure the characteristics of a torsional wave filter. Since the membranes are clamped rigidly at their peripheries we should expect the filter to be high-pass and the calculation of cos W indeed shows this to be the case. The same general result follows if the flexible membranes are replaced by solid plates. As one should expect, if the plates are free at their peripheries the structure acts like a low-pass torsional wave filter. Indeed when we pass to the limit of a plate of infinite rigidity, the structure acts precisely like the torsional wave filter described in the first part of this section. So far as the author is aware, no experimental results are available for this type of torsional wave filter.

4. *Filtration of Compressional Waves in Stratified Media*

Since any periodic nonhomogeneity in a material medium appears to result in selective elastic wave transmission, we should expect that a stratified medium consisting of alternate layers of two different substances will act as an elastic wave filter. In Fig. 8 is presented a schematic

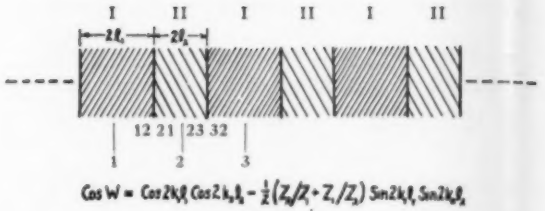

FIG. 8.

diagram of such an iterated structure of alternate layers of fluid media I and II with thicknesses $2l_1$ and $2l_2$, respectively. The densities of the media are, respectively, ρ_{01} and ρ_{02} with compressional wave velocities c_1 and c_2. Using the subscript notation indicated in the figure the boundary conditions for plane harmonic waves incident normally on the interfaces are simply

$$\begin{aligned} p_{12} &= p_{21}; \quad p_{23} = p_{32} \\ \dot{X}_{12} &= \dot{X}_{21}; \quad \dot{X}_{23} = \dot{X}_{32} \end{aligned} \tag{12}$$

and the application of these to the usual type of Eqs. (2) and (3) yield the pressure or particle velocity at mid-point 3 in terms of the corresponding quantities at mid-point 1, etc. The usual analysis (Section 1) then leads to a cos W expression of the following form[9]

$$\cos W = \cos 2k_1l_1 \cdot \cos 2k_2l_2 - \tfrac{1}{2}(Z_2/Z_1 + Z_1/Z_2) \sin 2k_1l_1 \cdot \sin 2k_2l_2. \tag{13}$$

Here $k_1 = \omega/c_1$ and $k_2 = \omega/c_2$. Moreover

$$Z_1 = \rho_{01}c_1/S, \quad Z_2 = \rho_{02}c_2/S$$

where S is the common area of cross section. Inspection of (13) indicates at once that the structure is a low-pass compressional wave filter. Certain general remarks may be made concerning the transmission and attenuation bands. For given media, the cut-off frequency between the first transmission and first attenuation bands varies more or less inversely with the section length. The same is true of the transition frequency between the first attenuation and the second transmission bands. For given section length, the two frequencies tend to increase as the acoustic resistances of the media approach each other, and their difference, of course, tends to become smaller. An illustrative cos W curve for a brass-water filter of this kind, with $l_1 = l_2 = 5$ cm, is given in Fig. 9.

The above treatment may be readily generalized to the case where the areas of cross section

differ for the two media. In fact the characteristics of the structure are given again by the cos W expression (13), where now, however, $Z_1=\rho_{01}c_1/S_1$ and $Z_2=\rho_{02}c_2/S_2$, if S_1 and S_2 are the cross-sectional areas of medium I and medium II, respectively. It thus appears that a tube with a succession of constrictions and expansions acts as a low-pass compressional wave filter (cf. Section 2 of Part I, Fig. 1).

The restriction to fluid media is unnecessary. One or both may be solid. However, the boundary conditions must now be altered somewhat. The pressure conditions in Eq. (12) remain the same but the volume current conditions must be replaced by equations stating the equality of displacement velocity on both sides of the interface. This leads to a cos W expression of the form

$$\cos W=\cos 2k_1l_1\cdot\cos 2k_2l_2 -\frac{1}{2}\left[\left(\frac{S_2}{S_1}\right)^2\frac{Z_2}{Z_1}+\left(\frac{S_1}{S_2}\right)^2\frac{Z_1}{Z_2}\right]\times\sin 2k_1l_1\cdot\sin 2k_2l_2. \quad (14)$$

This reduces to (13) for the special case in which $S_2=S_1$, but in general yields a different set of bands.[10] It is understood, of course, that $Z_1=\rho_{01}c_1/S_1$, etc. in (14).

A more general problem connected with stratified media is the transmission of *oblique* compressional waves through the structure. The detailed discussion will be presented elsewhere* and only certain general results will be mentioned here.

First taking up the case in which both media are perfect fluids, the boundary conditions (12) are modified only in the fact that the volume currents $\dot{X}$ are replaced by their components normal to the surface. For an infinite structure the characteristics are described by a cos W expression which can be thrown into the form (13) for the case of normal incidence, namely

$$\cos W=\cos 2Q_1\cdot\cos 2Q_2 -\tfrac{1}{2}(Z_2'/Z_1'+Z_1'/Z_2')\sin 2Q_1\cdot\sin 2Q_2, \quad (15)$$

where now, however,

$$Q_1=k_1l_1\cos\theta_1, \quad Q_2=k_2l_2\cos\theta_2 \quad (16)$$

and

$$Z_1'=\rho_{01}c_1/\cos\theta_1, \quad Z_2'=\rho_{02}c_2/\cos\theta_2. \quad (17)$$

In (16) and (17) θ_1 is the angle of incidence of the wave traveling from medium I to medium II, while θ_2 is the angle of emergence into medium II. From the law of refraction θ_1 and θ_2 are connected by the relation $\sin\theta_1/\sin\theta_2=c_1/c_2$. Incidentally (15) reduces to (13) when $\theta_1=\theta_2=0$. It is clear that the transmission characteristics of the structure depend on the angle θ_1 as well as on the acoustical properties of the media. We then meet a situation formally somewhat analogous to x-ray and electron diffraction by crystals: For a particular angle of obliquity, only certain frequencies will be transmitted or reflected as the case may be. An illustrative calculation has been made with water and ethyl ether as the two component fluids and shows that for all angles of incidence tested (from $\theta_1=20°$ to $\theta_1=75°$) the structure acts as a low-pass filter. With the increase in θ_1 the cut-off frequency for the first transmission band increases and the successive transmission and reflection bands are displaced to higher frequencies. Thus, for example, whereas for $\theta_1=20°$ the first transmission band extends to approximately 4200 cycles, for $\theta_1=75°$, the corresponding band extends to about 6200 cycles. At higher frequencies there is considerable overlapping of the bands for different angles. In general it turns out that the band width for given angle θ_1 remains constant.

When one of the two component media is a solid, the calculation of the transmission characteristics of the structure becomes materially more complicated because of the presence of both transverse and longitudinal waves in the solid layers. The procedure adopted by Reissner[11]

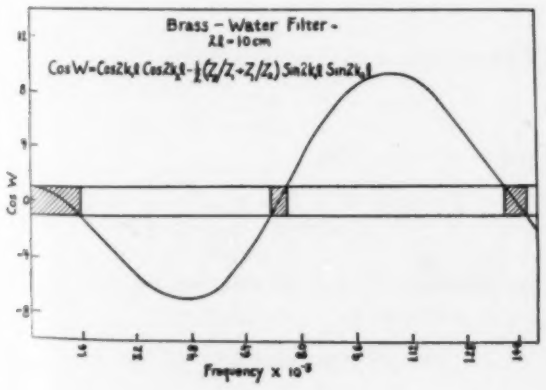

FIG. 9.

* Probably in the J. Acous. Soc. Am.

for the study of the oblique transmission of an elastic wave through a single plane parallel solid plate immersed in the fluid may be generalized with the result that the transmission is still given by a cos W expression analogous to (15), though rather more involved. For every angle less than the critical angle the result is a low-pass filter for compressional waves in the fluid separating the solid layers. There is apparently no record of the experimental study of such filters. A special case particularly interesting from a theoretical point of view is that in which the thickness of the solid layers is much less than that of the fluid layers, though not sufficiently so to render drum-head vibration of comparable order with genuine wave transmission. It then develops that cos W over a considerable range of low frequency can be written in the form

$$\cos W = \cos (2k_1 l_1 \cos \theta_1) + \epsilon(\nu), \tag{18}$$

where $\epsilon(\nu)$ is a periodic function of the frequency with an amplitude small compared with unity. Narrow reflection bands appear at those frequencies for which

$$2k_1 l_1 \cos \theta_1 = n\pi, \tag{19}$$

where n is integral. This is the elastic wave analog of the Bragg reflection law for x-rays.

5. *Acoustic Filtration in Dissipative Structures*

It seems worth while to close this article with a brief discussion of acoustic filters in which irreversible energy dissipation takes place. As was remarked in Part I, this has not been considered in our previous treatment.

The wave equation for plane waves in the x direction in a medium with dissipation coefficient α is

$$\ddot{\xi} + 2\alpha c\dot{\xi} = c^2 \frac{\partial^2 \xi}{\partial x^2}. \tag{20}$$

The solution for damped plane progressive harmonic waves of frequency $\nu = \omega/2\pi$ in both directions along the x axis can be put into the form

$$\xi = \xi_1 e^{-(\alpha+ik)x} e^{i\omega t} + \xi_2 e^{(\alpha+ik)x} e^{i\omega t}. \tag{21}$$

If, as usual, we employ the volume current $\dot{X}$ and excess pressure p as the fundamental acoustic quantities and denote their values at some chosen origin by $\dot{X}_1$ and p_1, respectively, we have for any other place in the medium

$$\begin{aligned} \dot{X} &= \dot{X}_1 \cosh (\alpha+ik)x - \frac{ik}{Z(\alpha+ik)} p_1 \sinh (\alpha+ik)x, \\ p &= p_1 \cosh (\alpha+ik)x + \frac{i(\alpha+ik)Z}{k} \dot{X}_1 \sinh (\alpha+ik)x. \end{aligned} \tag{22}$$

For the sake of illustration we shall restrict our treatment to the stratified medium at normal incidence. Let the dissipation coefficient for the first layer substance be α_1 and that for the second layer be α_2. If the notation otherwise remains the same as in Sections 1 and 4, the application of the usual boundary conditions leads to the following equations for p_{j+2} and $\dot{X}_{j+2}$ in terms of p_j and $\dot{X}_j$

$$\begin{aligned} p_{j+2} &= A p_j + \frac{i(\alpha_1+ik_1)}{k_1} Z_1 B \dot{X}_j, \\ \dot{X}_{j+2} &= A\dot{X}_j - \frac{ik_1}{(\alpha_1+ik_1)Z_1} C p_j \end{aligned} \tag{23}$$

with

$$\begin{aligned} A &= \cosh 2z_1 \cdot \cosh 2z_2 \\ &\quad + \tfrac{1}{2}(M + 1/M) \sinh 2z_1 \cdot \sinh 2z_2 \\ B &= \sinh 2z_1 \cdot \cosh 2z_2 \\ &\quad + (M \sinh^2 z_1 + 1/M \cdot \cosh^2 z_1) \sinh 2z_2 \\ C &= \sinh 2z_1 \cdot \cosh 2z_2 \\ &\quad + (M \cosh^2 z_1 + 1/M \cdot \sinh^2 z_1) \sinh 2z_2, \end{aligned} \tag{24}$$

where

$$z_1 = (\alpha_1 + ik_1)l_1, \quad z_2 = (\alpha_2 + ik_2)l_2$$

and

$$M = k_2 Z_1 / k_1 Z_2 \cdot (\alpha_1 + ik_1)/(\alpha_2 + ik_2).$$

Examination now discloses that

$$A^2 - BC = 1, \tag{25}$$

whence we can take

$$\begin{aligned} A &= \cosh W, \quad B = (B/C)^{\frac{1}{2}} \cdot \sinh W, \\ C &= (C/B)^{\frac{1}{2}} \cdot \sinh W. \end{aligned} \tag{26}$$

The usual method then enables us to write for p_{j+2} in terms of p_j

$$p_{j+2} = p_j e^W, \tag{27}$$

and a similar equation for $\dot{X}_{j+2}$ in terms of $\dot{X}_j$. Inspection shows that when $\alpha_1 = \alpha_2 = 0$, i.e.,

vanishing dissipation coefficients, $\cosh W$ reduces to the $\cos W$ expression in Eq. (14) (with $S_1=S_2$, of course). In this case, then, $\cosh W$ is real, and for $|\cosh W|\leq 1$, W is pure imaginary. On the other hand for $|\cosh W|>1$, W is real or complex with a nonvanishing real part. Hence in the limiting case of vanishing dissipation, the above analysis reduces to that of Section 4.

When α_1 and α_2 are nonvanishing, however, $\cosh W$ is complex and (27) indicates that there is no complete transmission for any frequency. This is to be expected when irreversible dissipation is in question. Nevertheless, if the dissipation coefficients are small, we can readily show that W will have the smallest real part for precisely those frequency ranges which are passed by the ideal nondissipative filter structure. Thus let

$$\begin{aligned} 2\alpha_1 l_1 &= p_1, \quad 2\alpha_2 l_2 = p_2 \\ 2k_1 l_1 &= q_1, \quad 2k_2 l_2 = q_2 \end{aligned} \tag{28}$$

and assume that for all frequencies considered $p_1 \ll 1$ and $p_2 \ll 1$. It then follows from (24) that

$$\begin{aligned} (\cosh W)_{\text{real}} &= \cos q_1 \cdot \cos q_2 \\ &\quad -\tfrac{1}{2}(Z_1/Z_2+Z_2/Z_1)\sin q_1 \cdot \sin q_2 \\ &\quad -p_1 p_2 \{\sin q_1 \cdot \sin q_2 \\ &\quad -\tfrac{1}{2}(Z_1/Z_2+Z_2/Z_1)\cos q_1 \cdot \cos q_2\} \\ &\quad -\tfrac{1}{2}(p_1 \cos q_1 \cdot \sin q_2 + p_2 \cos q_2 \sin q_1) \\ &\quad \times(\alpha_1/k_1-\alpha_2/k_2)(Z_2/Z_1-Z_1/Z_2) \end{aligned} \tag{29}$$

while

$$\begin{aligned} (\cosh W)_{\text{imag}} &= p_2 \cos q_1 \cdot \sin q_2 + p_1 \sin q_1 \cdot \cos q_2 \\ &\quad +\tfrac{1}{2}p_1 p_2(\cos q_1 \cdot \cos q_2 - \sin q_1 \cdot \sin q_2) \\ &\quad \times(\alpha_1/k_1-\alpha_2/k_2)(Z_2/Z_1-Z_1/Z_2) \\ &\quad +\tfrac{1}{2}(p_1 \cos q_1 \cdot \sin q_2 + p_2 \cos q_2 \cdot \sin q_1) \\ &\quad \times(Z_2/Z_1+Z_1/Z_2). \end{aligned} \tag{30}$$

Hence for all frequencies considered

$$|\cosh W|_{\text{imag}} \ll 1.$$

If we write $W=W_r+iW_i$, this means that

$$|\sinh W_r \cdot \sin W_i| \ll 1.$$

Hence either $|\sin W_i| \ll 1$ or $\sinh W_r \ll 1$ or both are much less than unity. Suppose the former is true and $\sinh W_r > 1$. Then necessarily $\cosh W_r > 1$. Since however

$$(\cosh W)_{\text{real}} = \cosh W_r \cdot \cos W_i,$$

this implies that $(\cosh W)_{\text{real}} > 1$. But from the form of $(\cosh W)_{\text{real}}$ in (29), the further implication is that in this case

$$|\cos q_1 \cdot \cos q_2 - \tfrac{1}{2}(Z_1/Z_2+Z_2/Z_1)\sin q_1 \cdot \sin q_2| > 1,$$

which is just the condition for an attenuation band in the ideal nondissipative structure (*Eq.* (14)). Now suppose alternatively that

$$|\sinh W_r| \ll 1.$$

Then $\cosh W_r \sim 1$ and $|W_r|$ is very close to zero. This can only happen, however, if

$$|\cos q_1 \cdot \cos q_2 - \tfrac{1}{2}(Z_1/Z_2+Z_2/Z_1)\cdot \sin q_1 \cdot \sin q_2| \leq 1.$$

Thus we see that the condition for a pass band in the ideal nondissipative structure is just the condition that W_r is very small in absolute value for the dissipative structure.

The introduction of irreversible dissipation thus does not seriously affect the position of the filtration bands. This conclusion to be sure depends on the lack of frequency dependence of the dissipation coefficient. If the dissipation is due to viscosity, for example, α varies as the square of the frequency and the above conclusion will presumably need modification at high frequencies.

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An Appeal For Fundamental Research*

By Herbert Hoover

THIS is a cheerful occasion. We are engaged in celebrating constructive action. Eternal fighting against destruction is depressing. For when you win, things are no better than they were.

At this time of confusion and discouragement in the political government of the world we should not neglect to observe the forces of individual achievement which are building steadily to improve the life of the common man. They are the rays of sunlight which penetrate the morning fog of this century. They are the real promise of a brighter day for all mankind.

The Training of Engineers

I have been asked to speak as an engineer upon the birth and purpose of a new Technological Institute.

The primary object of this Institute is to train engineers. I see it is part of the plan to place the boys in part-time overall jobs in industry during their training. That is an expansion of the method in other engineering institutions who require them to take overall jobs in summer. In any event when engineers leave college they mostly have to start for a cold, cold world in overalls. So that it is well to prepare them. Those overall jobs make for humility of spirit. They make for the common touch with fellow workers. They dilute technical arrogance with a little knowledge that all learning is not from books.

This experience also winnows out those who have made a mistake in choosing engineering with the notion that it is a silk-stocking profession. Those silk socks are the reward of making good.

And in all this emphasis on practical and training I wish to offer a word of caution and even protest.

These men will not deal wholly with physical forces. They will deal with human forces. Therefore their training must include a wide and a sympathetic understanding of the social sciences and the humanities.

I would like time to extend this discussion into this need of the social sciences and their application to the humanities. But this night, emphasis is given to the discussion of the new Institute of Technology and therefore the physical sciences.

Research in Physical Sciences

And I have something on my mind with regard to this new expansion of Northwestern. I have no doubt that officials in charge will do about as they please. But we on the outside of academic life should be allowed to have our say once in a while.

My suggestion goes far beyond training engineers in the way they should go. You will need build here great laboratories as the necessary class-rooms of engineering training. No institution of technology is worthy of its name that does not set part of its energies and laboratories to research in the physical sciences. And it is the research part that I wish to emphasize as equally vital to the future of our nation with training of engineers.

However, as Huxley insists, let me first define my terms. For the practical purposes of this discussion, I wish to distinguish between abstract or pure science research on one hand, and applied science research on the other.

Pure science research in this discussion means the search for fundamental natural law. That is the search for truth.

Applied science research is the application of such discoveries to practical use. That is invention. Pure science is thus the raw material of applied science. There is also another distinction. There is no big money for pure scientists. Sometimes they get an obituary notice fourth page inside.

I do not want you to think I am building a rigid wall between pure and applied science. Their purpose and borders are much blended. Our purpose must be to hold a balance within them.

* Principal part of an address by Dr. Hoover in celebrating the birth of the Technological Institute of Northwestern University.

Now you may be expecting me to advocate that research in these laboratories be directed mainly to applied science. That is what you would expect from the engineers, for our job is applied science. Quite the contrary, my hope is that the major emphasis be placed on pure science research. And the more abstract it is, the better. There are many reasons for all this.

Inadequate Support to Pure Science Research

We have in this country in industry and government probably 2000 laboratories of applied science. That compares with perhaps 300 before the war. They grab every discovery of fundamental law, and quickly turn it to invention and practical use. Including agricultural research they are supplied with probably $300,000,000 a year of income.

Our pure science research is dependent largely upon our universities. Including that portion of men and money which they can set aside plus that from all the other agencies it certainly does not amount to $20,000,000 a year. We allow pure science an expense account of about 5 percent of what we allow for cosmetics. Pure science research is therefore the poor relation but he furnishes the family wisdom.

There was a time when invention came from the starving genius in the garret. We got the steam engine that way. But invention is now too sophisticated to be wooed in these surroundings.

I can easily illustrate. Our electrical industries today are the by-product of the discovery of fundamental natural laws. They are the result of a half-century of research in the realms of pure physics and mathematics. It is from these realms that Faraday extracted the transformation of mechanical energy into electricity through induction. It is related that Gladstone was induced to visit Faraday's physics laboratory to see this new scientific contraption. When Gladstone, a practical man inquired whether it would be of any use to mankind, Faraday opined "I think some day you will be able to tax it." But it was long years afterward that Edison, Thompson, Siemens and the other engineers translated it into power and light. And today it moans and groans with taxation.

But the taxes are not its major achievement. Rivers of sweat have been saved from the backs of men. Watt and Faraday did more to reduce the 84-hour week to a 44-hour week than all the laws and all the organizations which have been embattled in this issue.

Infinite drudgery has been lifted from the hands of women. The electric light alone has relieved the human race from the curse of always cleaning oil lamps, scrubbing up candle drips, and everlastingly carrying one or the other of them about.

It adds cheer to life—enables us to postpone our spectacles for a few years longer. It has made reading in bed infinitely more comfortable. By merely pushing a button we have introduced the element of surprise in dealing with burglars. The goblins that lived in dark corners and under the bed have now been driven to the outdoors. The doctors now peer into the recesses of our insides. Our electric light enables our cities and towns to clothe themselves in gaiety by night, no matter how sad their appearance may be by day. It has lengthened the hours of our active lives and enabled us to read the type in the telephone book. It has become the friend of man and child.

But to return to our subject. From Faraday we move on to Maxwell's formulation of the electrical wave theory by pure mathematical deduction. Hertz confirmed Maxwell in experiment and proved the radiation of waves through space. It was the engineers Marconi and De-Forest who transformed these discoveries of pure physics into the radio communication of today. Out of that we get jazz by night and war scares by day. Between them it diverts our minds from other woes.

Again it was in the kingdom of abstract physics that Becquerel discovered radioactivity of certain substances and it was Curie who isolated radium. It was Dr. Kelly who applied this gift of abstract physics to the healing of human sores. So I could go on and on over the whole of modern technological advancement.

But my point is that these great gifts to men no longer come from the garret. Bread and water diet has been discarded by the mother of invention. These gifts come from long years of patient

experiment in great laboratories. Like the growth of plants cell by cell, of fact upon fact, some day there comes forth the blossom of discovery, the illuminating hypothesis or the great generalization. And finally it fruits into a multitude of inventions.

The Economic Importance of Pure Science Research

It has been the increasing productivity of men from these discoveries in pure science that has defeated the gloomy prophecies of Malthus on over-population. And if we are to maintain this civilization we must have more and more discovery in natural law. Daily we develop labor saving devices. At once men are thrown out of jobs. That partially corrects itself because products are made more cheaply, consequently more are consumed and more men are employed. Nevertheless we have a constant stream of technological unemployment. That stream can only be reabsorbed by new industry producing new commodities and services. We can be sure of these new industries, if our pure science research is feeding its raw materials into the hoppers of the applied science laboratories. This does not need to be some revolutionary advance such as electricity or the gas engine. It may be made of a thousand comparatively minor discoveries and advances.

And let me inject here another observation. These advances can come alone among free men. They must be free to choose their own callings, pursue their own devices. They come alone in a society which conserves the dignity and individuality of men. The soil of discoveries is intellectual and spiritual liberty. The soil of their application to human use is the initiative and enterprise of free men.

Neither collectivist action nor government has ever yet discovered a fundamental law of nature.

There is no price in money that the world could not afford to pay today to these rare men whose imagination, whose intellectual honesty, whose infinite patience carry abstract science with these great strides. They, like frontier explorers of a century ago, are not interested in mere money rewards. Yet one such great explorer in science is worth more in money to the country than all the deposits in our banks. The training of such men, the providing of facilities for them to live and work with are a part of your job if you are to take your place with our other great Institutes of Technology. If you, like Massachusetts Institute of Technology, can find and flower a man like Karl Compton, or like California Institute of Technology can give scope to a man like Robert Millikan, then you will have become a great institute. Then young men inspired with the spirit of adventure on these new frontiers will flock to your halls.

The Opening of New Frontiers to Youth and Progress

And that brings me to another phase. We have been told with monotonous repetition by the collectivists and left-wingers that our frontiers are gone. They say our industrial plant is built. They claim there is no safety valve for human energies. They assure us that we have come to an age of humdrum problems of underconsumption, over-production, and the division of the existing pot. They say that new opportunity for youth has shrunken.

That is not so. There was never in history a more glorious frontier to youth than today. Adventure and opportunity beckon in every avenue of science. They beckon from the great profession of men trained to research. They beckon from its thousands of applications. From it spring tens of thousands of new services and industries. In them human courage, character, and ability have an outlet that never came even with the two-gun frontiers. Just as the new villages followed the stockades of the frontier, so do new cities follow every new mastery of technology and power.

It has only been 40 years since the gas engine came into its own. Today more people are supported in the automobile and aviation industries and their collateral services than ever had an outlet through our original land frontiers. And they enjoy a far higher standard of living and comfort. For adventure, for personal achievement, it has every glory those frontiers ever had. And let those who lament the loss of frontier life not forget the adventures with the traffic cop

that emanate from the discovery of the combustion engine.

On a smaller scale, almost yesterday, the profession of geophysics took from the pure science of earthquake laboratories a whole method of determining undisclosed geology of the earth. This application has doubled our oil reserves. Five thousand young engineers have jobs, adventure, and advancement from it. The price of gasoline has been cheapened by it, and we go on more and more narrow escapes. And who can say that these 25 million automobiles do not face the adventure of sudden death a dozen times a day.

And there are more frontiers opening before us this very day. The last ten years have seen the unfolding of new knowledge of atomic structure which influences all chemistry. We know of cosmic rays. We labor to explain the Einstein theory. These momentous discoveries are still in the chrysalis stage of pure science. These new expansions of the frontiers of fundamental law in physics will yet produce new and startling inventions. They will open new areas of adventure, new occupations, and new comforts to humanity.

Conclusion

The product of scientific research is not alone the building of industry. It is not alone the eradication of disease nor the multiplication of harvests. It is not alone the extension of opportunity and adventure.

It is a certainty that research into the fundamental laws of nature contributes to the moral and spiritual life of mankind. Here lie the unfolding of beauty, the ever-widening of the boundaries of knowledge. Here is the "inculcation of veracity of thought" in a world sodden with intellectual dishonesty. Here is the lifting of men's minds beyond the depressing incidents of the day. Here is the confirmation of a Supreme Guidance in the universe far above man himself.

Our civilization is much like a garden. It is to be appraised by its blooms. We must fertilize its soil with liberty. We must be diligent in cultivation. We must be constant in expansion of its frontiers. We must guard it against destructive forces. Then will we produce more and more of those blossoms the fragrance of whose lives stimulates increased endeavor, gives widening opportunity to youth and courage for the future.

The Principles of Statistical Mechanics*

THE statistical theory of the properties of matter is now on a firm foundation, and is sure to find increasing use in the applied physicist's study of materials. Not only does it provide ways of calculating thermodynamic properties from spectroscopic data, a method which has been extensively developed in the past decade, but it also gives a deeper insight into the meaning of the thermodynamic functions than is provided by the more abstract approach from the first and second laws. Moreover, the statistical method gives the only approach to the problems of molecular kinetics—the calculation of time rates of approach to equilibrium from non-equilibrium starting conditions.

In spite of all this the teaching of statistical mechanics has been singularly incomplete and backward. The result is that the methods and point of view of the statistical theory have not been used nearly as generally as they should be. To our shame we must confess that the chemists have been more enterprising than the physicists in developing the applications of statistical mechanics.

Professor Tolman's book deals with the *principles* of statistical mechanics and only includes an occasional application by way of an illustrative example. In the preface he says

* Richard C. Tolman, *The Principles of Statistical Mechanics* (Oxford University Press, New York, 1938), pp. 661+xix, 16×24 cm. Price $9.00.

frankly, "Applications to actual physical-chemical systems are included in the present book only insofar as is desirable for illustrating the nature of the fundamental principles of statistical mechanics and the methods to be employed in applying them. The needs for a complete account of the manifold applications of statistical mechanics to physics and chemistry should be met for a long time by the exhaustive treatment given by Fowler in the second edition of his *Statistical Mechanics*."

The central aim of the work is stated thus: "Throughout the book, although the work of earlier investigators will not be neglected, the deeper point of view and the more powerful methods of Gibbs will be taken as ultimately providing the most satisfactory foundation for the development of a modern statistical mechanics." It should be emphasized that this is an entirely new book and in no sense a revision of Professor Tolman's earlier book on statistical mechanics.

This book, like Fowler's is, I am afraid, apt to be too difficult for all but the ablest students unless they are especially interested in critical analysis of the foundations. Thus we are still without a book suitable for an elementary approach although this situation promises to be remedied soon by the book on statistical mechanics just written by Professor Slater.

The book is divided into three main parts: First, 174 pages, is a thorough, painstaking and critical presentation of the classical theory. Especially valuable is Tolman's critical analysis of the fundamental Boltzmann H theorem which provides the justification of the intuitive belief that a system will tend toward the most probable state in course of time. Second, 341 pages, is devoted to the quantum-statistical mechanics. Of these the first 144 pages are devoted to an account of the elements of quantum mechanics along quite familiar lines. This subject matter has been dealt with so thoroughly in a wide variety of treatments, e.g., Dirac, Kemble, Rojansky, Pauling, and Wilson, that it is unfortunate that Tolman takes the space to tell the same story again. The book would have been immeasurably improved by extreme condensation of this portion and substitution of more illustrative material from the field of applications. Third, 136 pages, is devoted to the relation of thermodynamics to statistical mechanics which is the sole *raison d'être* of the subject so far as physics is concerned. Kinetic applications are not included.

The critical analysis of the H theorem in both classical and quantum statistics is the outstanding new contribution of the volume. In addition we are given a presentation of fundamentals which is often thrilling. Chapter IX on the use of the density matrix for discussing statistical assemblies is especially satisfying and should go a long way toward giving a wider understanding of this important and too much neglected contribution of von Neumann and Dirac.

Theoretical physicists will find Tolman's book a welcome addition to the literature, but I am afraid the graduate students will find it rather discouraging as an introduction unless they do some judicious skipping around. Somewhat in the same spirit as the *New York Herald-Tribune* once published a guide to Aldous Huxley's *Eyeless in Gaza*, let me suggest that a student who has "had" elementary quantum mechanics and the usual descriptive courses in atomic physics would find it more profitable to do the book in this order: Chapters 1, 2, 3, 4; 7 and 8 as review of quantum mechanics; 9, 10, 13 and 14. This will give him a good survey of the field with omission of the more difficult H theorem work which can be studied separately in chapters 5 and 6 (classical) and 11 and 12 (quantum) and which should be worked on in close parallelism for best results.

E. U. Condon
Westinghouse Research Laboratories

American Institute of Physics

Symposium on Temperature, Its Measurement and Control in Science and Industry

Hotel Pennsylvania, New York, New York, November 2, 3 and 4, 1939

PLANS for this previously announced* Symposium have now been advanced so far by the energetic work of the committees that some details of the program may be published. Although the program is not yet complete, the authority of the authors and the wide scope of the papers listed below fully assure the outstanding value of the Symposium. The number of inquiries received regarding the program and participation in it indicate that there will be a large attendance.

Registration and Printed Program

As was announced before, the Symposium is open to all interested persons and they are cordially invited to attend. An extensive printed program, containing titles, authors and full abstracts of papers to be presented will be ready for distribution in October. This program will serve as an informative advance outline of the Symposium, as a serviceable guide to the sessions themselves and as a permanent condensed record of the proceedings. A supply will be printed and sent in answer to requests until the edition is exhausted. Anyone may insure his receiving this program in advance by registering for the Symposium by mail enclosing the fee of one dollar. Opportunity to register will also be provided at the meeting but *advance registration will greatly facilitate all arrangements for the service and convenience of those who plan to attend.* (Address: Temperature Symposium Committee, American Institute of Physics, 175 Fifth Avenue, New York, New York.)

Dinner

A special banquet with refreshments preceding it and an interesting program following it is being arranged for the evening of Friday, November 3. This will provide a pleasant social occasion for the better acquaintance of those attending the Symposium.

Publication

Many of the papers presented will be published later in journals of the Institute (including *The Review of Scientific Instruments* and the *Journal of Applied Physics*) or in other appriate journals. It is also planned to arrange for the publication in book form of a complete record of all the valuable contents of papers and discussions.

Tentative List of Papers

Automatic Control of Laboratory Furnaces by the Wheatstone Bridge Method by H. S. Roberts

Changes in Platinum Thermocouples Due to Oxidation by Bert Brenner

Cooling of Concrete Dams by Douglas McHenry

Correction of Real Gas Scales to the Scale of an Ideal Gas by C. S. Cragoe

Description and Response of Constantan-Silver Plated Thermopiles (Wound 104 Turns per Inch) by L. M. K. Boelter and J. Gier

Determination of Humidity from Temperature Observations by A. W. Ewell *et al.*

Diurnal Winter Fluctuation of Soil Temperatures Under Bare Surface, Under Cover Crop and Under Orange Tree by F. A. Brooks and Coby Lorenzen, Jr.

Electric Dewpoint Recorder by R. H. Reed

Experimental Comparison of Pyrometric Methods for Liquid Iron in the Foundry by Earnshaw Cook

Factors Affecting the Life of Platinum Thermocouples by C. F. Homewood

Field of Extreme Temperatures by W. M. Cohn

Gas-Temperature Measurement by H. F. Mullikin

Geologic Temperature Recorders (Note: These "recorders" are not man-made recorders) by Norman L. Bowens

Heat Measurement by Thermal Analysis of Metals by Cyril Stanley Smith

High Speed Temperature Recording Apparatus by G. K. Brokaw and A. R. Champion

* See Journal of Applied Physics, March 1939 and Review of Scientific Instruments, May 1939.

High Velocity Thermocouple for Gas-Temperature Measurement BY H. F. MULLIKIN

Iron-Nickel Alloys Suitable for Resistance Thermometry BY M. A. HUNTER

Joule-Thomson Coefficient in the Realization of the Absolute Temperature Scale BY J. R. ROEBUCK

Logical Statement of the Principle of the Thermodynamic Scale BY C. H. PRESCOTT, JR.

Man's Bodily Reactions to His Thermal Environment BY C. E. A. WINSLOW

Measurement of High Temperatures in Gases by the Sound Velocity, Spectroscopic, and Density Methods BY C. A. SUITS

N. B. S. Temperature Scale Below the Oxygen Point BY HAROLD J. HOGE

New Two-color Optical Pyrometer BY H. W. RUSSELL, C. F. LUCKS AND L. G. TURNBULL

Optical Pyrometry BY W. E. FORSYTHE

Performance Characteristics of Recording Potentiometers BY C. O. FAIRCHILD

Potentiometers for Precision Measurements BY W. P. WHITE

Practical Course in Temperature Measurement BY P. H. DIKE

Principles of Automatic Temperature Control BY C. O. FAIRCHILD

Production and Measurement of Temperatures Below 1°K BY CHARLES F. SQUIRE

Progress of Standardization of Temperature Measurements by The British Standards Institution BY J. G. BENNETT

Pyrometric Cones (3 papers) BY G. A. BOLE *et al.*

Pyrometric Uses of the Platinum Metals BY F. E. CARTER

Pyrometry in Connection with Creep Tests BY F. H. NORTON

Pyrometry in the Manufacture of the Platinum Metals BY F. E. CARTER

Pyrometry in the Preparation and Application of Tin BY C. L. MANTELL

Pyrometry of the Iron Blast Furnace BY G. R. FITTERER

Pyrometry of Liquid Metals BY G. R. FITTERER

Pyrometry of Liquid Steel with Molybdenum-Tungsten Thermocouple BY JOHN CHIPMAN

Pyrometry of Liquid Steel in the Open-Hearth Furnace BY L. O. SORDAHL

Pyrometry of Oxide-Coated Filaments BY C. H. PRESCOTT, JR.

Radiosondes in Obtaining Upper Air Temperatures BY CHRISTOS HARMANTAS

Relation of Composition to E.m.f. in Various Alloys Suitable for Thermoelectric Pyrometry BY M. A. HUNTER

The Relation of Uniform Pyrometer Charts to Uniform Products BY J. A. DOYLE

Reproducibility of the Ice Point BY J. L. THOMAS

Resistance Thermometry BY E. F. MUELLER

Simple Constant Temperature Source for Checking Optical Pyrometers BY C. F. LUCKS AND H. W. RUSSELL

Simple Electronic Temperature Controller BY W. T. REID

Stability of Base Metal Thermocouples BY A. I. DAHL

Stellar Temperatures BY G. P. KUIPER

Summary of Papers of Institute of Fuel, Symposium on Temperature Measurement BY J. G. BENNETT AND M. PIRANI

Surface Temperatures (Measurement of) BY F. C. HOUGHTON

Temperature and Temperatures Scales BY H. T. WENSEL

Temperature Distribution, Extremes, and Trend Tendencies over the Earth's Surface BY J. B. KINCER

Temperatures in Atomic Phenomena BY F. L. MOHLER

Temperature Measurements by Interferometry BY R. B. KENNARD

Temperature Measurements by Special Methods in Use at the Smithsonian Institution BY C. G. ABBOT

Temperature Measurements in Low Temperature Calorimetry BY J. G. ASTON

Temperature Measurements in Molding Plastics BY J. DELMONTE

Temperature Measurements in the Pottery Industry BY EDWARD SCHRAMM

Temperature Measurements with Probes of Large Mass in Stoker Fuel Beds BY MARTIN A. MAYERS

Temperature of Man in Health and Disease BY E. F. DUBOIS

Temperature Sense in Man BY H. C. BAZETT

Theoretical Study of the Design of Thermocouples for Experimental Cracking Units BY E. R. BROWNSCOMBE

Thermal E.m.f. of Various Alloys BY J. M. LOHR

Thermocouple Anemometer BY W. U. HUKILL

Thermocouples for Steam Temperature Measurement BY B. O. BUCKLAND AND S. S. STACK

Thermoelectric Measurement of Low Temperatures BY R. B. SCOTT

Thermometry in Hygrometric Measurements BY A. W. EWELL

Thermoelectric Thermometry BY W. F. ROESER

Upper-Air Temperatures and Their Significance in Air Mass Analysis BY H. R. BYERS

Use of Isotherm Data in Corrections of Gas Scales BY F. G. KEYES

COMMITTEES

The foregoing remarkable program of papers on the measurement and control of temperature amply confirms the wisdom of those whose initiative led to its preparation. It also abundantly demonstrates that there is a widespread interest in the subject and a general recognition of its fundamental importance in science and engineering. The Institute feels that the general

purposes for which it sponsored the Symposium will be well served.

It has taken an unusual amount of time and energy on the part of the committees, particularly the Symposium Committee, to plan and develop the broad structure of the program. Members of the Committee have acted as Chairmen of sub-committees in different fields. In carrying out their responsibilities they have taken great pains to secure the cooperation and advice of persons well fitted through training and present interests to supplement their own familiarity with their respective fields. The American Institute of Physics is indebted to them and their associates.

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Cooperation from Others

Although the American Institute of Physics is primarily responsible for the arrangement of the Symposium, it could not have been so successful with the program without much cooperation from other groups. It is particularly indebted to:

The National Research Council
and the
National Bureau of Standards

and to officers and committees of many societies, including

American Ceramic Society
American Chemical Society
American Institute of Mining and Metallurgical Engineers
American Society of Mechanical Engineers
American Society for Metals
American Society of Refrigerating Engineers
American Society for Testing Materials
American Standards Association
American Welding Society
Society of Automotive Engineers

Here and There

Cover Photograph

The cover photograph shows a transmission pole being ripped to pieces while being struck by a 3,600,000-volt artificial lightning bolt. The photograph was made in the high voltage laboratory of the Westinghouse Electric and Manufacturing Company's transformer division at Sharon, Pennsylvania, during a demonstration of lightning surges.

New Appointments and Promotions

Dr. S. J. Simmons, who has been associated with the Radiation Laboratory at the University of California at Berkeley, California, has accepted a position as Senior Research Fellow at the University of Pittsburgh. Mr. David Halliday has been appointed Junior Research Fellow. Their work will be in the field of nuclear physics and is supported in part by the Buhl Foundation of Pittsburgh.

★

Word comes from Professor J. W. Butcha, Chairman, Department of Physics, at the University of Minnesota that Dr. Lynn H. Rumbaugh has been promoted to an Associate Professor.

★

Dr. Bruno Rossi has been appointed research associate in the Department of Physics of the University of Chicago, where he will work with Professor Arthur H. Compton. The appointment has been made possible by a grant from the Emergency Committee in Aid of Displaced Foreign Scholars. Dr. Rossi was deprived of his professorship at the new Physical Institute of Padua by the government's decrees of September, 1938. For three years he had been in charge of planning and building the new institute and equipping it for research in atomic and nuclear physics.—*Science*, Sept. 1, 1939.

★

Professor George F. Uhlenbeck arrived from Europe on the *Queen Mary* August 30 and is resuming his duties as Professor of Physics at the University of Michigan after an absence of several years.

★

Dr. Frank Rasetti, Professor of Spectroscopy at the University of Rome, will be at Laval University in Quebec, Canada, this year.

★

The University of Leeds has conferred the degree of Doctor of Science on Dr. W. L. Bragg, Cavendish Professor of Physics at the University of Cambridge.

Calendar of Meetings

October	
5–7	American Institute of Mining and Metallurgical Engineers, Coal Division, Columbus, Ohio.
12–14	Optical Society of America, Lake Placid, N. Y.
13–14	Society of Rheology, National Bureau of Standards, Washington, D. C.
23–25	American Institute of Mining and Metallurgical Engineers, Metals Division, Chicago, Ill.
23–27	American Society for Metals, Chicago, Ill.
November	
2	U. S. Institute for Textile Research, New York, N. Y.
2–4	American Institute of Physics, Symposium on Temperature: Its Measurement and Control in Science and Industry, New York, N. Y.
3–4	Acoustical Society of America, Iowa City, Iowa.
2–4	American Institute of Mining and Metallurgical Engineers, Industrial Minerals Division, Tuscaloosa, Ala.
13–17	American Petroleum Institute, Stevens Hotel, Chicago, Illinois.
15–17	American Institute of Chemical Engineers, Providence, R. I.
December	
1–2	American Physical Society, Chicago, Ill.
4–9	17th Exposition of Chemical Industries, Grand Central Palace, New York, New York.
27–Jan. 2	American Association for the Advancement of Science, Columbus, Ohio.
28–30	American Physical Society, Columbus, Ohio.
28–30	Geological Society of America, Minneapolis, Minn.

New Books

Heat Engines. A. C. WALSHAW. Pp. 413+xv, Figs. 251, 18.3×12.2 cm. Longmans Green and Company, London, 1938. Price $2.75.

This book is written as an introductory text for students who are preparing for first examinations in applied thermodynamics and heat engines. The presentation assumes only an elementary knowledge of physics and mathematics. The subject matter is treated in three main divisions. Part I presents a survey of fundamental physical concepts and their application to heat engines. Part II deals particularly with the steam engine, with special chapters devoted to such topics as boilers, condensers and valves. Part III deals with internal combustion engines and has chapters on air compressors and fuels.

There are in all nineteen chapters with illustrative problems at the end of each. The solutions of these problems are included. The text is profusely illustrated with about two hundred and fifty line drawings and sectional sketches. These figures are an essential aid in presenting the operating characteristics of the various machines. The descriptions in the main are extremely lucid. Typographically there is little to be desired.

It might reasonably be objected that the index is too brief to be adequate. With the rapidly expanding use of the compression-ignition engine one might have hoped for a more generous treatment of this subject in a text appearing at this time. No references to original papers are included.

Throughout the book the c.h.u. (centigrade heat unit) is employed as a unit for heat energy. To American engineers the use of this heat unit is perplexing, to say the least. If they must give up the widely used irrational B.t.u. (British thermal unit), then it is surely much better to adopt the joule or calorie.

These criticisms are not of a serious nature and the author must be commended for the very clear presentation of an important subject.

J. M. CORK
University of Michigan

Light. F. BRAV. Second edition. Pp. 369+xii, Figs. 245, Plates 8, 12½×18½ cm. Longmans, Green and Company, New York, 1939. Price $2.50.

This is the second edition of an excellent elementary textbook by the late science master at Clifton College in England. The historical viewpoint, with frequent but cogent criticism, makes the book more than a compendium of optical facts. The student cannot help but acquire some idea as to how science has developed in the past and by what methods it goes forward now. A knowledge of algebra and trigonometry is assumed; in fact, calculus is used in a few derivations, which are, however, set up in finer print. This enables the author to give an effective presentation of such topics as minimum deviation, achromatization, and the resolving power of instruments and of spectroscopes of the prism and grating type. A considerable number of simple, instructive experiments are thoroughly explained. Problems are given at the end of nearly every chapter, the answers being placed at the end of the book. Extra problems are also given there. This text seems to be very well suited for the optics part of a two-year course in general physics.

JOSEPH VALASEK
University of Minnesota

The Electrochemistry of Gases and Other Dielectrics. GEORGE GLOCKLER AND S. C. LIND. Pp. 469+xiii, Figs. 82, 15×23 cm. John Wiley & Sons, New York; Chapman and Hall, London, 1939. Price, $6.00.

The Committee on Electrical Insulation of the National Research Council has for a long time been interested in the deteriorating effect of corona and gaseous electric discharges on organic insulation. It was at the request of this Committee that Professors Glockler and Lind brought together in a thoroughly commendable way the literature on chemical reactions in electrical discharge. Both of the authors have contributed extensively to the experimental and theoretical sides of the field. In preparing this treatise they exceeded the hopes of the Committee in that they have covered a wider scope than anticipated. They have treated the subject of electrical activation in all forms of discharges involving numerous types of reactions.

The book is divided into three parts. In Part I, typical reactions in various forms of discharge, including cathode rays, are illustrated with examples from the literature. The formation of molecular fragments such as free radicals, atomic hydrogen and oxygen are also discussed. Part II deals with numerous reactions in which the reactants are arranged in order of increasing atomic or molecular weight. An interesting discussion of the nitrogen afterglow is given. Part III concerns itself more with the theoretical aspect of the subject. Electron affinity of atoms and molecules, ionization during chemical reaction and mobility of ions are treated. A chapter each is devoted to cathodic sputtering of metals and a mechanism of chemical reaction in electrical discharge.

Although the writing of the book was inspired by an engineering group it will probably be of most interest to physicists and chemists concerned with gas discharges. It should be a very valuable reference book as over 1300 published papers are reviewed. The material is presented in a thoroughly scientific manner. This work represents the first attempt to bring some order out of seemingly unrelated experiments which lie on the borderline of physics and chemistry. In spite of the fact that the literature is extensive and that the work dates back to the beginning of experimental science, no unifying theory has resulted. Thus, finality of theory is not to be expected in this

volume. Much of the work was undoubtedly inspired by the lure of fascinating syntheses. Much more must be done before the complicated phenomena involved can be thoroughly understood.

L. J. BERBERICH
Westinghouse Research Laboratories

Patent Tactics and Law. ROGER SHERMAN HOAR. Pp. 315+xv, 14½×21 cm. The Ronald Press Company, New York, 1939. Price $4.50.

The title well describes the contents of the book. Tactics used in handling various types of difficult situations pertinent to patent matters are fully explained and interspersed with a maximum of patent law and a minimum of legal phraseology so that the subject matter may be digested with relative ease by the layman.

The purpose of the book is to make available, for industrial executives and engineers, information concerning patent matters which such men require if they are fully to understand and properly to assist their patent attorneys in the work of building up a patent structure adequate for the protection of the interests of their company.

Topics such as evaluation of patents, what is patentable, preliminary protection, interferences, infringements, searches, validity, legal papers, etc., are treated fully and clarified by many practical examples and concise definitions. Illustrative of the definitions we submit:

> A patent is the grant by the government of a seventeen-year right to exclude others from making, using or selling anything which the courts may hold to be within the purview of any one of such claims of the patent as may be held to be valid.

Mr. Hoar consistently wages war on misconceptions. The above-quoted definition, for instance, banishes the erroneous belief that a patent grants the right to make, use, and vend the patented article.

This book is a revision of Mr. Hoar's volume *Patents*. (Ronald Press, 1916.) Those familiar with the earlier work will find that the revision contains 22 chapters instead of 18, an increase of 64 pages in the text, a rearranged order of chapters and subject matter, corrections regarding fees, time intervals, etc., for conformity with the latest patent office rulings, nearly a thousand footnotes (entirely lacking in the earlier volume) consisting of cross references, references to supplementary material, explanations and illustrative cases; and finally, an appended table of 15 pages wherein the illustrative cases, are arranged alphabetically by title, each with identification as to source and with the number of the page where it is used as a reference.

The reviewer feels that *Patent Tactics and Law*, has a broader field of usefulness than that for which it was originally intended. It is deserving of careful study by all laymen who are confronted with problems concerning patents.

G. R. GREENSLADE
Research Laboratory, Flannery Bolt Company, Bridgeville, Pennsylvania

Photography by Infrared. Its Principles and Applications. WALTER CLARK. Pp. 397+x, Figs. 101, 15½×23½ cm. John Wiley & Sons, New York, 1939. Price $5.00.

The author is a member of the research staff of the Eastman Kodak Company. In this volume he has brought together the widely scattered information dealing with the many phases of infra-red photography. The purpose of the volume is to furnish a guide for the practical infra-red photographer, regardless of the aspect of the subject he is concerned with, be it commercial, artistic, scientific, or technical. A great deal of material has been reviewed and the useful and interesting parts woven together into what is a very valuable reference book. As such it is quite complete even to the inclusion of well-known or obvious information on the theory and practice of photography in general.

The use of infra-red rays to obtain unusual effects in landscape photography and to record distant objects has been noted by everyone. However, the precise effects of haze, smoke, and fog on infra-red visibility is not commonly known. This book contains a good practical summary of theoretical and experimental studies in this field.

One is impressed by the widespread scope of the useful applications of infra-red photography. Some of the fields benefited by this development are: spectroscopy, astronomy, the examination of paintings and documents, studies of the subcutaneous venous system, leaf structures and pigments, and microphotography of various objects and thin-sections.

The volume contains a valuable discussion of the characteristics of various sources of infra-red radiation, of filters, and of a large variety of materials of particular interest to the infra-red photographer. Moreover, tables listing some of the properties of available infra-red photographic supplies, data for converting weights and measures, and formulae for processing solutions are included. All this, together with an extensive bibliography of the literature dealing with the various aspects of this subject make the book contain just about everything a photographer of the infra-red might need to know; except the best exposure time. However, the information given will greatly help the photographer in making some pretty good guesses.

JOSEPH VALASEK
University of Minnesota

Contributors to This Issue

W. C. Elmore received a B.S. in Engineering Physics from Lehigh in 1932 and his doctor's degree from Yale, 1935. He is now a member of the Physics Department of Swarthmore College.

John P. Blewett received his B.A. and M.A. degrees at the University of Toronto in 1932–33, and his Ph.D. at Princeton University in 1936. He is now associated with the Research Laboratory of the General Electric Company at Schenectady.

J. R. Pierce received his science degrees and his Ph.D. from the California Institute of Technology during 1933–36. He is now a member of the Technical staff of the Bell Telephone Laboratories and is engaged in the development of various types of vacuum tubes.

Maurice L. Huggins was born in Berkeley, California in 1897. He received the Ph.D. degree at the University of California in 1922. After holding a National Research Fellowship at Harvard and the California Institute of Technology, he joined the chemistry faculty at Stanford and, later, Johns Hopkins. Since 1936 he has been a research chemist with the Eastman Kodak Company.

Carl Kenty's picture and biography appeared on p. 36 of Vol. 9, No. 1, 1938.

C. G. Suits' picture and biography appeared on p. 187 of Vol. 10, No. 3, 1939.

R. B. Lindsay's picture and biography appeared on p. 630 of Vol. 9, No. 10, 1938.

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Max Auwärter received his doctor's degree in philosophy after studying at the Universities of Tuebingen and Munich. At present he is head of the physical and metallographic laboratories of the W. C. Heraeus Company at Hanau, Germany.

W. C. Elmore

J. P. Blewett

M. L. Huggins

J. R. Pierce

Contributed Original Research

The Viscosity of Dilute Solutions of Long-Chain Molecules

III. The Staudinger Viscosity Law*†

MAURICE L. HUGGINS
Kodak Research Laboratories, Eastman Kodak Company, Rochester, New York
(Received March 22, 1939)

An equation has recently been derived for the specific viscosity of a dilute solution of "randomly-kinked" chain molecules, which, for long chains and small concentrations, is of the same form as Staudinger's empirical "law": $\eta_{sp}=k_s nc$. The validity of this equation for actual solutions, and the nature and magnitude of the deviations to be expected are discussed. Among the effects considered are those due to restriction of rotation about the bonds in the chain and to the size and nature of the solvent molecules, of the repeating units in the chain, and of the end groups.

AS is well known, the specific viscosity of a solution—that is, the relative increase in viscosity over that of the pure solvent—is a quantity which depends greatly on the structure of the solute molecules. Generalizing and extending a method due to Werner Kuhn,[1] a general equation giving this dependence in terms of the coördinates of the atoms in the solute molecules has been derived and reported in the previous papers of this series.[2] This equation may be written

$$\eta_{sp}=\frac{\eta}{\eta_0}-1=\frac{k_E v/V+6\pi Nc'(F_{zz}')_{Av}}{1-6\pi Nc'(F_{zz}')_{Av}}, \qquad (1)$$

with

$$F_{zz}'=\frac{\Sigma a_i r_i^2\cos^2\theta_i\cdot\Sigma a_i r_i^2\sin^2\theta_i}{\Sigma a_i r_i^2}. \qquad (1a)$$

Here η and η_0 are the viscosity of the solution and of the pure solvent, respectively; v/V is the volume of solute molecules per unit volume of the solution; N is Avogadro's number; c' is the concentration in moles per cc; k_E is a small constant, depending on the relative sizes and shapes of the solute and solvent molecules; a_i is the effective radius of atom i; and r_i and θ_i are the projections of polar coördinates of atom i on the plane which includes the direction of shear and the direction of maximum velocity gradient. The quantity F_{zz}' is to be averaged over all the molecules.

Applying this general equation to the hypothetical case of a rodlike molecule, composed of n spherical "submolecules" of effective radius a, uniformly spaced with a distance l between their centers, an equation has been obtained, which, for large n and low concentrations, simplifies to

$$\eta_{sp}=\frac{\pi}{24}Nl^2an^3c'=\frac{\pi}{24{,}000}Nl^2an^2c, \qquad (2)$$

where c is the concentration in "submoles" per liter.

Similarly, the following limiting equation has been obtained for chain molecules which are kinked—with a uniform bond angle but otherwise in a random way—at each submolecule:

$$\eta_{sp}=\frac{\pi}{3\sqrt{5}}Nl^2aB_\infty n^2c'=\frac{\pi}{3000\sqrt{5}}Nl^2aB_\infty nc. \qquad (3)$$

B_∞ is a symbatic function of the bond angle $(\pi-\alpha)$, which has the value 2 for "tetrahedral" bond angles (Fig. 1).

Comparison with experimental data[3, 4] for solutions of normal paraffins shows much better

* Communication No. 713 from the Kodak Research Laboratories.

† Presented at the Pittsburgh Meeting of the Society of Rheology, December 28, 1938.

[1] W. Kuhn, Zeits. f. physik. Chemie **A161**, 427 (1932).

[2] For the first two papers in this series, see J. Phys. Chem. **42**, 911 (1938); **43**, 439 (1939).

[3] H. Staudinger and F. Staiger, Ber. **68**, 707 (1935).

[4] K. H. Meyer and A. van der Wyk, Helv. Chim. Acta **18**, 1067 (1935).

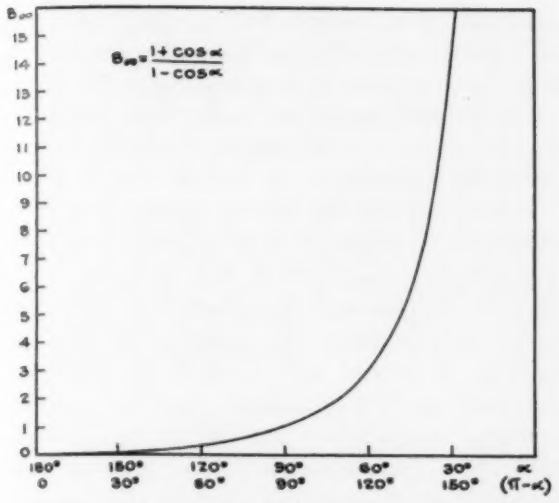

FIG. 1. Variation of B_∞ with bond angle $(\pi-\alpha)$.

agreement with the kinked molecule equation than with that for a straight molecule (Fig. 2).

The limiting kinked-molecule Eq. (3) is identical in form with the equation

$$\eta_{sp}=k_s nc, \tag{4}$$

which Staudinger[5] has applied to many long-chain substances, although the new theoretical interpretation differs markedly from his.[6]

[5] H. Staudinger, *Die hochmolekularen organischen Verbindungen* (Julius Springer, Berlin, 1932); also Zeits. f. Elektrochem. **40**, 434 (1934); Helv. Chim. Acta, **19**, 204 (1936) and many other journal articles.

[6] The idea that these long-chain molecules are kinked in a more-or-less random way is of course not new, and comparison of the theoretical equations of Kuhn and others for rod-like molecules with Staudinger's empirical law has been taken by K. H. Meyer, H. Mark (Zeits. f. Elektrochem. **40**, 449 (1934)), and by Kuhn himself (Kolloid Zeits. **68**, 2 (1934); Angew. Chem. **49**, 858 (1936)) as evidence for the kinked molecule picture. Kuhn has derived statistically expressions for the root-mean-square distances between the first and last atoms in a long randomly-kinked chain molecule, between the middle atom and the straight line joining the first and last atoms, and between the atom one-quarter of the distance from one end of the chain and the plane containing the first, last and middle atoms. Considering as important (for the calculation of the specific viscosity) only the Einstein term $(5v/2V)$, he then uses for the average volume per solute molecule the volume of an ellipsoid having axes of length equal to the root-mean-square distance just mentioned. (Actually a large proportion of this volume is occupied by *solvent* molecules, and the proportion varies with the length of chain.) This leads to an equation showing proportionality between η_{sp}/c and $n^{\frac{1}{2}}$. The exponent of n in this equation can be increased somewhat by introduction of a factor, of arbitrary form and containing an arbitrary constant, to allow for the space-filling effect. A strong argument against this procedure is that the space-filling constant would necessarily vary greatly with the size of atoms and groups attached to the chains, whereas, as Staudinger has shown, the exponent of n is close to unity for a wide variety of long-chain compounds, having various side-chains.

The more exact equation, which has been derived for randomly-kinked molecules, is

$$\frac{\eta_{sp}}{c}=\frac{\dfrac{k_E M_0}{1000\rho}+\dfrac{7\pi N B_\infty \beta l^2 a(n-2+1/n)}{45{,}000}}{1-\dfrac{7\pi N B_\infty \beta l^2 a(n-2+1/n)c}{45{,}000}}. \tag{5}$$

M_0 is the "molecular weight" of a submolecule, e.g., 14 for CH_2; ρ is the density of the pure liquid solute; and β is a function of n and of the bond angle, which approaches unity as n increases (Fig. 3).

The chief purpose of this paper is to discuss the validity of the assumptions involved in the derivation of this equation and in its application to actual solutions.

Equation (5) was first derived, without the first term in the numerator, on the assumption that each submolecule is an incompressible sphere, past which the surrounding liquid flows at the velocity it would have at the sphere center, if the other submolecules were not near to interfere with and deflect the streaming of the surrounding solution. It is obvious that several assumptions are involved here which are not in entire agreement with the actual state of affairs in solutions in which we are interested. However, a group of atoms in a submolecule may be replaced, hypothetically, by a single spherical atom producing the same frictional energy loss. Moreover, the effect of neighboring submolecules is primarily a shielding of a portion of the surface from "contact" with the moving solution, making the true energy loss due to each submolecule a certain fraction (independent of the position of that submolecule in the chain, unless it is at or near one end) of what it would be if the submole-

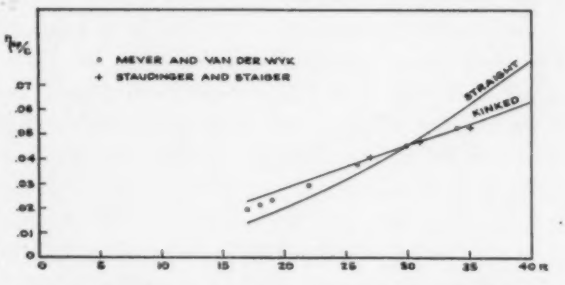

FIG. 2. Comparison of experimental and theoretical values for specific viscosities of normal paraffin solutions.

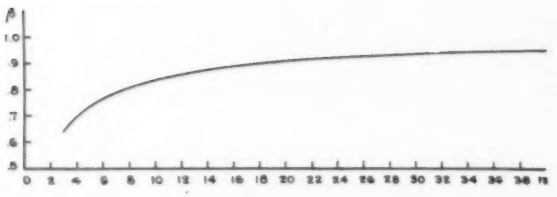

FIG. 3. Variation of β with n for tetrahedral bond angles.

cules were all separated from each other by considerable distances. For actual cases, therefore, one may either introduce a constant factor, independent of n but depending on the size and shape of the submolecules and on their distance apart, or else define a as the "effective radius" of the submolecule, i.e., the radius of the hypothetical spherical atom which would give the same viscosity contribution, if bathed in a liquid whose motion was undisturbed by neighboring submolecules. The magnitude of a can be roughly estimated for simple cases, especially if its value for another not-too-different chain molecule is known. It is best determined empirically, however, from measurements of the viscosity of solutions of appropriate molecules of known length.

This procedure does not take account of the fact that the solution surrounding a given submolecule is not moving with the same velocity at all points of that submolecule's "surface"—if we may use the term—as was assumed in the derivation. The correction for this should be the same, regardless of whether the submolecules are linked together into a chain or are moving independently in the solution. It can be shown that, if the submolecules are similar in size and in force field to the solvent molecules, no correction term need be added. In other words, k_E in Eq. (5) is zero. This is necessary to make the viscosity of a solution of such (sub) molecules which are not linked together equal that of the pure solvent. For very large spherical submolecules, k_E must equal 5/2, to give the result calculated by Einstein[7] for solutions of (sub)molecules of this sort, not linked together. For most cases of chain molecules, it is probably sufficiently accurate to assume that k_E is not far from zero and that the term containing it can be neglected in comparison with the other term. This neglect doubtless introduces some error for very short chains. (It may, for instance, be responsible for the deviations between Meyer and van der Wyk's viscosity data for the shorter paraffins and the theoretical curve. See Fig. 2.)

In deriving Eq. (5), it was assumed that the distribution of orientation of the chain molecules is random—a result of random collisions with other molecules—rather than one in which orientations parallel to the direction of shear are favored, as would be expected if Brownian motion collisions were absent. Calculations made by Kuhn,[1] by Boeder,[8] and by the writer[9] show that this assumption is quite accurate for relatively small molecules, such as the paraffins already considered, at ordinary temperatures with ordinary rates of shear. For very large molecules or low temperatures or high rates of shear it is less accurate. However, if the molecules are randomly-kinked, with tetrahedral bond angles, little error is introduced. Calculations for the extreme case of no Brownian motion lead to an equation of the same form as before, with the constant only 4 percent less.[2] The dependence on temperature and rate of shear, for large molecules, is greater the larger the bond angle, i.e., the more extended the chain, on the average. Rodlike molecules, with no Brownian motion, would tend to line up parallel to the direction of shear and the second term in the numerator of Eq. (5) would be close to zero.

The calculations so far discussed all assume rigid molecules. For molecules in which there is a degree of flexibility, resulting from the possibility of partial or complete rotation about the bonds of the chain, the picture is changed somewhat, but the numerical results, for temperatures, rates of shear and types of molecule of most interest, are not changed markedly. The shearing force causes the molecules to rotate, all in the same direction. During two quadrants of the rotation, a flexible molecule is being extended—stretched—and its contribution to the viscosity energy loss is increasing. During the other two quadrants of rotation, it is becoming more compressed and its viscosity contribution is diminishing. Except

[7] A. Einstein, Ann. der Physik [4], **17**, 549 (1905); **19**, 289 (1906).

[8] P. Boeder, Zeits. f. Physik **75**, 258 (1932).

[9] M. L. Huggins, unpublished calculations.

under extreme conditions the effects of stretching and contraction nearly cancel each other.[9]

The question of the randomness of the kinking in actual molecules is an important one. In the calculations which have been made, there has been no allowance for the fact that some molecular configurations would produce overlapping of atoms. It can be shown, though, that this neglect makes but little difference, for most chain molecules of interest, at least. On the average,

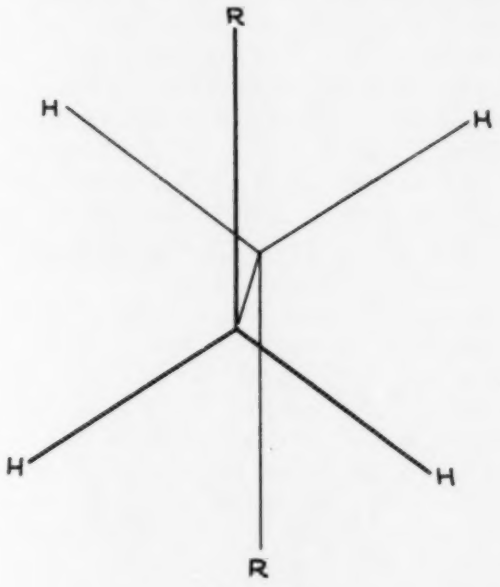

FIG. 4.

the molecules are slightly more extended than according to the calculations, hence the viscosity is slightly greater on this account. The error in the constants due to this cause is doubtless much less than errors due to other approximations made in the derivation.

It is now known that rotation about a single bond in a paraffin chain is not free but is very much restricted, on account of interaction between the groups at the two ends of the bond.[10] The potential curve for rotation of one CH_2R group relative to the other has 3 maxima and 3 minima. At ordinary temperatures there may be oscillation about the orientations corresponding to the 3 minima, but only rarely over one of the maxima. Little is known quantitatively about these maxima and minima. If the three minima were all alike, i.e., if the three stable orientations had equal energy, the statistical distribution of configurations of long-chain molecules would closely approximate that which has been calculated for completely random kinking—that is, for equal probability of all orientations at each single bond. Actually, the 3 minima are certainly not alike. If, for example, a minimum corresponding to the orientation shown in Fig. 4 were considerably lower than the other two (obtained by rotation of one CH_2R group through 120°), the average molecule would be more extended than if the minima were all alike, and the specific viscosity would be higher. The nature of the dependence on chain length would not be altered however; for long chains the effect would be practically equivalent to that of increasing the "effective radius" a. If a is determined empirically therefore, this source of uncertainty in the theory can be neglected.

There is some evidence[11] indicating that in the paraffins the three minima correspond to the orientations shown in Fig. 5. If so, similar remarks to those in the preceding paragraph apply, although a preference for one orientation would produce less effect on the viscosity. Corresponding considerations must also be applicable to chain molecules other than the paraffins.

It is obvious that the nature of the atoms or groups attached to the atoms in the chain must greatly affect the degree to which the molecules are extended or coiled. In general, mutual attractions between these atoms or groups favor coiling and hence lower viscosities, while intramolecular repulsions favor the more stretched-out configurations, hence higher viscosities. However, one must be careful in applying this generalization, as there are doubtless many

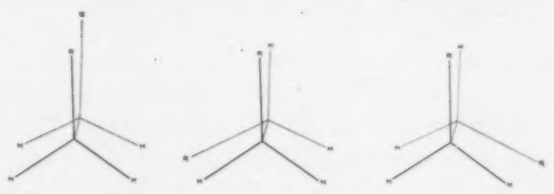

FIG. 5.

[9] J. D. Kemp and K. S. Pitzer, J. Chem. Phys. **4**, 749 (1936); K. S. Pitzer, J. Chem. Phys. **5**, 473 (1937); J. D. Kemp and C. J. Egan, J. Am. Chem. Soc. **60**, 1521 (1938); G. B. Kistiakowsky, J. R. Lacher and W. W. Ransom, J. Chem. Phys. **6**, 900 (1938).

[11] E. Bartholomé and J. Karweil, Zeits. f. physik. Chemie **B39**, 1 (1938); G. B. Kistiakowsky, J. R. Lacher and W. W. Ransom, reference 10.

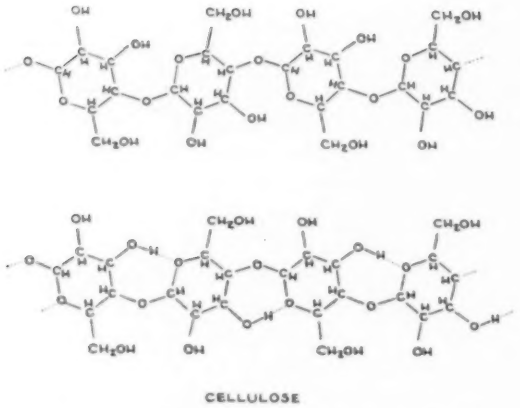

FIG. 6. Illustrating the way in which hydrogen bridges may stabilize the extended zigzag chain in cellulose.

exceptions. For example, there is reason to believe that chains of fully esterified cellulose tend to be much more kinked than chains of cellulose itself. In the latter, one would expect quite strong attractions between the hydroxyl hydrogen atoms and the ether oxygen atoms—a tendency to form OHO hydrogen bridges.[12] Instead of making the molecules coil up more than they would otherwise, however, these bridges are probably responsible for the extended zigzag chains, which x-ray data have shown to exist in the fiber (Fig. 6). In solution, extended chains of this sort produce much higher specific viscosities than if the chains were irregularly kinked.

Considerations of the effect of the nature of the solvent on the specific viscosity are somewhat similar. If the solvent molecules are strongly attracted by the atoms or groups all along the solute chain, they may, by steric hindrance, make the more coiled-up configurations impossible or improbable and so favor the more extended configurations. This increases the specific viscosity. (Strong attractions of this sort also increase the effective radius a, but this, in the writer's opinion, is usually of less importance.) On the other hand, strong mutual attractions between solvent molecules, like strong attractions between different parts of a solute molecule chain, favor the coiled configurations and so low viscosities.

In most chain molecules of interest, the groups at each joint are not all alike, even neglecting those at the ends of the chain. Vinyl polymers, for example, contain alternately CH_2 and CHR groups. Then, too, the lengths, l, of the segments between joints may not all be alike. In some instances also, the bond angles may have two or more values. If the distribution of different side-chains, lengths, and bond angles is approximately uniform, it seems reasonable to substitute, in Eq. (5), average values for $B_x\beta$, l^2 and a. This procedure, of course, does not take account of the factors disturbing the randomness of the kinking, which have just been discussed.

For relatively small molecules, the effect of the end groups of the solute chains is sometimes not negligible. A partial correction may be made by averaging $B_x\beta$, l^2 and a. The true correction, however, would be expected to be somewhat larger, since the greater the average distance of an atom from the center of gravity of the molecule, the greater is its contribution to the specific viscosity. A more accurate correction can doubtless be made, but hardly seems worth while at this time.

The theoretical considerations which have been presented in this paper must, of course, be tested by critical comparison with experimental data. Although this has to some extent been done by the writer, using data in the literature, a thorough discussion of these data and their implications would necessarily be lengthy and will not be attempted here. Work on the problem of obtaining more and better pertinent data is now in progress in these laboratories.

[12] M. L. Huggins, J. Org. Chem. 1, 407 (1938).

Rhodium Mirrors for Scientific Purposes

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(Received March 8, 1939)

DIFFERENCES in quality between the precision mirrors of optical instruments and the more usual looking glasses create actual differences in kind so great that entirely different methods must be used in their manufacture. Lately the method of cathode atomization ("sputtering") has been successfully used in making optical mirrors and their quality in continued use has been actually enhanced by making the reflecting film of rhodium, a metal having a lower but a far more constant reflecting power than the more usual silver.

The manufacture of mirrors by applying a tin amalgam coating to the back of a piece of glass goes back to the 14th and 15th centuries. Much later (1843), the method of depositing metallic silver by reduction of silver nitrate solution was developed and much improved by Liebig. Both the amalgam and reduction methods are used today to make looking glasses and mirrors of a like quality using gold, platinum, palladium, copper and nickel, as well as silver.[1]

These layers are applied to the back of the glass, which thus serves to protect their reflecting surfaces from corrosion, and not to the face of glass as is necessary in precise optical mirrors. The application of the metal coat to the precisely shaped surface of a piece of glass or other amorphous material required in optical instruments presents a more difficult problem since it is necessary that the metallic layer be extremely thin and uniform and that it be able to resist corrosive influences of itself without a supplementary protective coating.

The only method for applying these layers available until quite recently was chemical reduction followed by polishing of the thin metal coating. The polishing operation is likely to detract from the quality of the mirror for optical purposes.

In the course of development in this art fundamentally new methods were adopted, the first and most generally practiced of which was that of cathodic atomization ("sputtering"). Broadly, this method consists in heating a cathode on which is impressed a high voltage, e.g. 1000–4000 volts, in a vacuum of 0.01 to 0.5 mm of Hg. The temperature is raised by ionic bombardment to so high a point that portions of the cathode are heated above the melting point and vaporized. This vapor is deposited in the form of a thin metallic layer. In the early stages of the development lack of sufficiently powerful sources of current made it impossible to treat large cathode surfaces and hence to produce large mirrors. Later, electric generators became available which supplied adequate energy at high voltage and permitted large electrodes to be used, so that relatively large mirrors could be produced. Because of the comparatively large quantities of gas in the atomizing space, and in view of the marked tendency of metals in an extremely finely divided state to absorb large quantities of gas, the coatings obtained in this way did not have the ideal physical properties of compact metal.[2]

A further method of preparing mirror coatings is based on readily decomposable noble metal salts which after a burning process leave the metal behind in the form of a mirror coating. This method, however, has the great drawback that the glass supports have to be heated to so high a temperature that their quality from an optical point of view is impaired. These difficulties were eventually overcome by methods utilizing volatilization of electrically heated fine filaments in vacuum such as are met in incandescent electric lamps.

The main contribution to this development was made by Pohl and Pringsheim in the year 1912.[3] Their method consisted in heating the

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[1] L. Winkler and R. Ernst, *Die Spiegelfabrikation* (A. Hartleben, Vienna, 1936).

[2] E. O. Hulbert, Rev. Sci. Inst. **5**, 85 (1934).

[3] R. Pohl and P. Pringsheim, Verhandlungen der Deutschen Phys. Ges. **14**, 506 (1912).

metal in a crucible to a temperature above its melting point and utilizing the higher vapor pressure thereby set up to produce the mirror coating rapidly. This method has maintained its supremacy up to the present day with slight variations, namely the use of tungsten boats or tungsten coils. Hochheim, in particular, has developed this method on a very large scale during the last ten years.

However, only in the last four years has this method been fully successful, particularly for the production of surface mirrors. John Strong[4] first made use of this method on a large technical scale to coat astronomical mirrors with aluminum. Methods of manufacturing on a large scale by the vaporizing process are given by L. Dunoyer.[5] The method of the author differs from the usual method principally in important details.

The importance of the new methods lay in the possibility of replacing backed mirrors with surface mirrors in precision optics, but in the early development particular interest centered about the choice of metals to give the most favorable results. Silver, the classical material for producing mirrors, was predominantly used at first. However, as the art of precision optics made more exacting demands, particularly as to constancy of reflecting power, silver no longer sufficed.

Of the possible metals to replace silver, the first and most obvious to try, owing to its highly reflecting qualities, was aluminum. Since this metal has serious drawbacks, especially in mechanical properties, efforts in this direction were concentrated on the aluminum alloys, particularly those with silver, silicon and magnesium. Platinum, whose reflecting power is considerably lower than that of silver and aluminum, was largely used to make weatherproof mirrors, in spite of its high price and in spite of its unfavorable optical properties. Numerous attempts to improve the mechanical properties of aluminum and aluminum alloys by the use of intermediate layers show how far they fall short of the ideal mirror coating.[6]

Only in recent years a new metal of the platinum group, namely rhodium, has been used for mirrors. Its reflecting power is 15 percent below that of platinum, but only eight percent below that of aluminum and aged silver. For mirrors it possesses distinct advantages as will be seen.

It might be imagined that the best plan of development would have been to seek as high a reflecting power as possible. This was by no means the case. On the contrary, the direction of development has been dictated by other considerations and has proceeded from the use of silver with the highest reflecting power, to aluminum with a somewhat poorer reflecting power, and finally to rhodium with a yet poorer reflecting power. The principal reason for this is to be found in the aging which occurs in thin layers of metal and which the newer methods of mirror manufacture emphasized.

Experience has shown that the reflecting power of a new mirror is always greater than after it has been stored or in use for a considerable length of time. Fig. 1 shows the curves for the reflecting power of silver, aluminum, platinum, silicon, and rhodium in the fresh state. Silver has by far the greatest reflecting power of all these metals. Aluminum, whose reflecting power is independent of wave-length

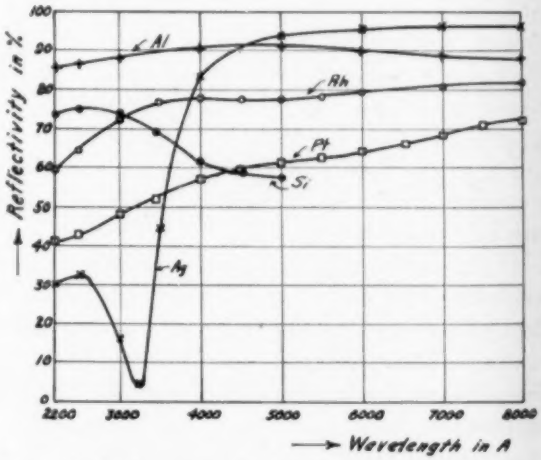

FIG. 1. Reflecting power of Ag, Al, Si, Pt, and Rh in the fresh state.

[4] John Strong, Astrophys. J. **83**, 401–423 (1936); John Strong and E. Gaviola, Phys. Rev. **2**, 207 (1936); J. Opt. Soc. Am. **26**, 153–162 (1936).

[5] L. Dunoyer, Comptes rendus **202**, 474–76 (1936). See also W. C. Heraeus G.m.b.H., Zeits. f. Instrumentenkunde **57**, 221 (1937); M. Auwärter, Zeits. f. tech. Physik **18**, 457 (1937), and "Die Umschau" **43** (1937).

[6] R. C. Williams, Astronomical J. **77**, 316 (1933); Phys. Rev. **41**, 255 (1932); **46**, 146 (1934).

up to a point far beyond the visible portion of the spectrum, has a reflecting power which is seven percent lower than that of silver. Platinum and silicon are different. Whereas silicon has great reflecting power mainly in the upper portion of the ultraviolet, its reflecting power falls off considerably in the visible range and only amounts to 40–50 percent at wave-lengths between 4000 and 8000A. Of all the metals considered, rhodium's reflecting power is nearest to that of silver and aluminum. The rise of its curve in the visible spectrum amounts to approximately four to five percent between the wave-lengths 4200 and 7800A which for practical purposes is not perceptible to the eye. The sensitiveness of the eye is such that the rise must be at least seven to eight percent to produce perceptible coloring.

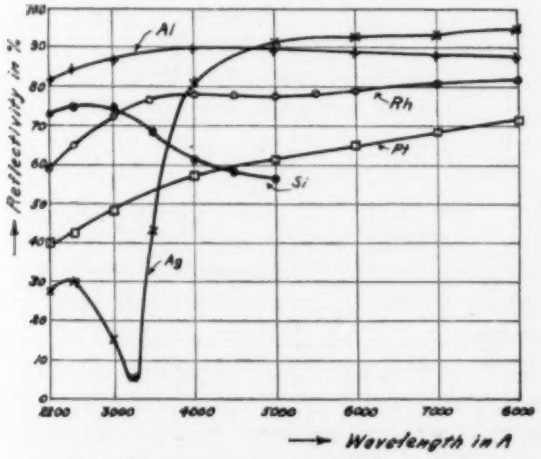

FIG. 2. Reflecting power of Ag, Al, Si, Pt, and Rh in physically aged state.

The aging process has the effect of altering the reflecting power curves of almost all metals, both in relative position and in relation to wave-length. Two causes for aging must be taken into consideration separately. One form is purely physical, while the other involves a chemical change in the layer.

Physical aging alters the structure of the metal. When freshly applied, the metal layer has a loose and not perfectly regular structure. In the course of time, the layer becomes noticeably harder with the formation of a practically regular grain. The curves of the above mentioned metals in a physically aged state are plotted on Fig. 2.

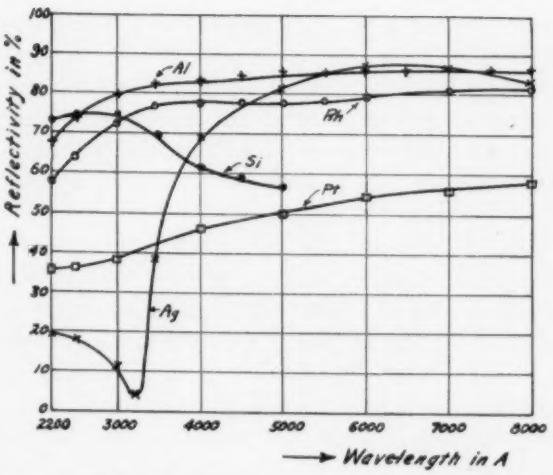

FIG. 3. Reflecting power of Ag, Al, Si, Pt, and Rh in chemically and physically aged state.

Generally speaking, chemical aging has a worse effect than physical change upon the qualities of the mirror. From a chemical point of view, the surface of the mirror coating combines with certain residual gases present in the atmosphere to form compounds which are themselves colored. Since light in being reflected must pass twice through the portion thus affected (in reaching the metal and coming away from it), it undergoes an intensified selective absorption. Curves representing chemical aging are plotted in Fig. 3. These curves show that silver more than other metals produces a pronounced yellow reflection. This is due to the fact that the chemically changed superficial portion of the coating absorbs the red and blue sections of the spectrum to a marked extent. This change naturally causes a reduction in the reflecting power, so that, as may be seen from the curves, the reflecting power of silver drops to a point below that of aluminum and only slightly above that of rhodium.

The curves show that all the metals with the exception of rhodium are subject to both physical and chemical aging. The need to avoid such changes, particularly in precision optical instruments used for carrying out absolute measurements, is obvious. Herein lies the great advantage of the new rhodium mirror which shows a degree of constancy hitherto unattainable.

A further troublesome feature of mirror coat-

ings is their lack of resistance to temperature, particularly in the presence of air. A silver coating becomes quite useless at temperatures between 180° and 200°C, but a rhodium coating may be used, even for long periods of time, at temperatures up to 430°C without any appreciable loss of reflecting power.

As a consequence of both physical and chemical aging, scattering of light occurs with the majority of mirror coatings.[7] The reason for this is that the surface of a mirror has a granular structure and therefore acts as a diffraction grating, especially for light of short wavelengths. The consequence is that images reflected by such a mirror are not sharp, but are blurred, a particular drawback in astronomy. In this respect, too, rhodium coating represents a remarkable advance.

Attempts have been made to inhibit chemical aging by a separately applied protective layer. For this purpose both organic and inorganic materials have been tried. The simplest procedure is to cover the freshly prepared coating with a thin skin of pyroxylin lacquer. This method, however, has the drawback that the coating is thinner at the middle than at the edge and consequently has a certain distorting effect on the optical image. Varnishing of astronomical mirrors is quite inadmissible since the thickness of the protective layer gives each point of light a circle of diffraction. Protection by inorganic substances is equally imperfect. Here, we distinguish between three different procedures. One consists in the use of thin glass foils of a few hundredths of a mm thick, a compromise which obviously limits the uses to which mirrors thus prepared can be put. A second method consists in the application of thin coverings of quartz by evaporation in a high vacuum, but the results obtained experimentally by the authors and others show that this method is of little practical value for the reason that the protective layers thus obtained generally produce marked interference colors. The third method has disadvantages which are much less objectionable than those of the two methods already described. It depends upon the valuable property of aluminum, of combining with oxygen to yield clear, colorless compounds in the form of thin, extremely dense films. The thickness of such a film, which is capable of resisting weak acids, amounts to only approximately 1/10,000 of a mm. Such aluminum oxide skins do not resist chlorine and sulfate if present in the atmosphere. Moreover, they are extremely sensitive to alkalies. Dew, particularly in boggy districts, has a harmful effect upon aluminum coatings even when protected in this way. Acid gases absorbed by the dew react with the mirror coatings.

The only previously known mirror coating that does not require protection is that made of platinum, but even this has serious drawbacks. Platinum coatings tested for invariability in a tropical chamber have shown pronounced discoloration even after a relatively short time. Although the reason for this could not be at once ascertained, the cause is probably the formation of compounds with gases. Their presence has been definitely proved recently in the course of investigations carried out for another purpose.[8]

Rhodium is the only mirror metal known today which retains its reflecting power unaltered for any length of time without the use of a protecting layer and irrespective of the nature or condition of the surrounding atmosphere or medium.

The protective layers described above are designed to fulfill the further purpose of protecting soft metals from mechanical damage. However, the slightest injury to the protective layer, particularly on a silver mirror, soon leads to complete spoiling of the mirror since chemical agents can reach the silver coating through the damaged portion of the protective layer. The behavior of rhodium is unique in this respect also. Its hardness is so great that mirrors coated with rhodium can be cleaned with soft cloths without special care having to be taken. There is no danger of the mirror becoming scratched, as there generally is with all other mirror coatings, particularly when organic protective layers are used.

A further respect in which rhodium is markedly

[7] R. Swany, Proc. Ind. Acad. Sci. (A) 1, 347 (1934).

[8] M. Auwärter and K. Ruthardt, Zeits. f. Elektrochemie 44, 579–585 (1938).

superior to other mirror metals is its extraordinary adhesion to the material of the support. The inferior adhesion of aluminum and silver becomes apparent as soon as the mirror coating is damaged at any point, since the coating soon becomes detached from the support, mainly under the influence of water, with the result that the mirror becomes unfit for further use. No instance of this has ever been observed with properly prepared rhodium mirrors.

Semi-Transparent Mirrors

The mirror coatings so far considered are intended to reflect incident light as completely as possible. Other important mirrors have the property of allowing part of the incident light to pass through unchecked and unmodified and of reflecting a further part in a form as far as possible unmodified. The sum of the transmitted and reflected light should approximate as nearly as possible the intensity of the incident light. The development of these so-called filters encountered the most unexpected difficulties. In the optical art recourse was first had to forms of construction whose properties could solve only part of the problem presented.

The first method is to reduce the light beam in a definite ratio. For this purpose the law of distance is employed when the apparatus permits, and where this is not possible, use is made of the rotating sector which screens off a certain predeterminable portion of the light while allowing a certain portion to pass unchecked. The beam thus produced is consequently not a continuous but an interrupted flow of light. That this expedient can only be adopted as a last resort for most uses will be clear.

A great advance was made in the field of light reduction by the application of the principle of polarization. Two Nicol prisms make it possible to reduce light intensity to any desired extent. This result is achieved with the disadvantage that the transmitted light is no longer like the incident light, but is linearly polarized.

Subsequently smoked glasses, in which colloidally dispersed particles of soot are incorporated in the glass, make it possible to obtain the transmitted light in approximately the same physical form as the incident light. Here, the dispersion of light by the individual impervious particles diffuses parallel incident rays in their passage through smoked glass and the optical image obtained is consequently impaired.

The next substantial improvement in the quality of light reducing filters was achieved by the production of extremely thin platinum foils which transmit and reflect certain amounts of the incident light. These filters were particularly valuable as compared with those made of other metals, since the transmitted light is let through with practically the same factor of reduction irrespective of its wave-length throughout the entire visible range. Initially white light thus reappears after traversing the filter as white light of reduced intensity.[9]

Quite unexpectedly, results obtained with rhodium show a considerably higher degree of constancy of the transmitted light irrespective of wave-length than with platinum. While platinum foil of various thicknesses shows troublesome fluctuations of transparency, the behavior of rhodium is equally independent of wave-length at all thicknesses. Moreover, while platinum filters always have to be covered by protective films to counteract atmospheric influences, this precaution is unnecessary with rhodium.

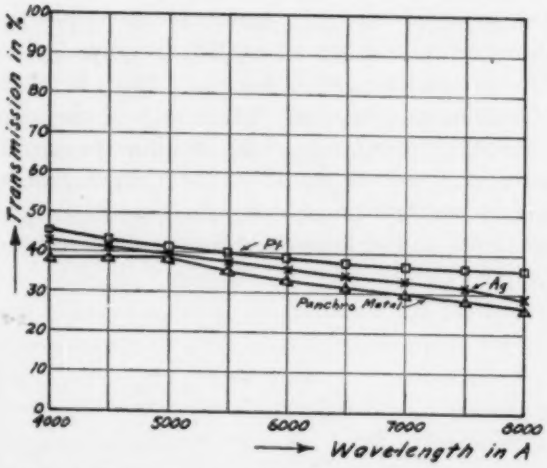

Fig. 4. Transparency of Ag, Panchro Metal and Pt after aging.

[9] Hagen and Rubens, Ann. d. Physik **8**, 432 (1902); Dorgelo, Zeits. f. Physik **31**, 827 (1925); Artigas, Rev. d'optique **5**, 217 (1926); Kienle, Zeits. f. Astrophys. **1**, 13 (1930); Schuch, Ann. d. Physik **13**, 270 (1934); Auwärter, Zeits. f. tech. Physik **18**, 437 (1937); and Die Umschau, **43** (1937).

Even in the thin layers necessary for semi-transparent mirrors, the adhesion of rhodium is so great that with reasonably careful handling the coating will not be damaged. Extremely thin coatings of this kind are best cleaned by careful wiping with a soft pad.

A peculiar feature of extremely thin metallic foils and coatings is that they acquire a certain degree of cloudiness in the course of time. This cloudiness is only superficial and is probably due to surface adsorption. It is possible to remove this film of dirt completely by careful cleaning without in any way affecting the properties of the mirror. This incidentally proves that the film of dirt is quite independent of the coating itself, for if the metal coating were even slightly affected by chemical influences it would be bound to have a different degree of transparency after cleaning.

Dispensing with protective films on semi-transparent mirrors is of great importance, since it eliminates the danger of selective adsorption of light by this film. The complete absence of circles due to the scattering of light is also a valuable point in favor of using thin metal layers as filters. The fact that rhodium does not age is a further advantage in semi-transparent mirror coatings as in opaque ones. Whereas aluminum is useful for totally reflecting mirrors, the transparency of this metal is so dependent on wave-length as to render it quite useless for semi-transparent mirrors. Fig. 4 shows the transparency of platinum, silver, and "Panchro" metal (an Mg-Al alloy frequently used as a mirror metal in the United States). Fig. 5 presents graphically the transparency of platinum and rhodium in different thicknesses of foil. These curves show clearly the advantages of rhodium over other metals used for coatings of this kind. The new technique developed by the firm of W. C. Heraeus G.m.b.H. for coating mirrors and for producing semi-transparent mirrors allows surfaces of an area up to 1 sq. m to be given a highly uniform rhodium coating of any desired degree of transparency within wide limits. This firm is actually able to guarantee accuracy within ±five percent over large surfaces and within ±one percent over small surfaces, limits not hitherto attainable.

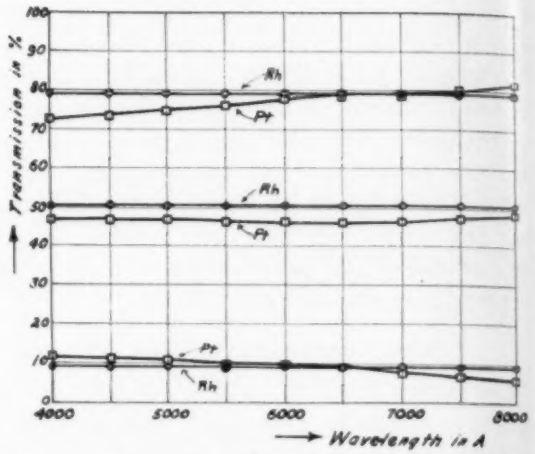

FIG. 5. Transparency of Pt and Rh for different thickness of foil after aging.

In this way it has become possible to produce semi-transparent mirrors with the precision necessary for use in large instruments. New applications where large surfaces are required, mainly as light reducers and also as stepped wedges and tapered filters, promise excellent performance from the new coatings of semi-transparent mirrors with rhodium.

The Motion of Cylindrical Particles in Viscous Flow

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(Received April 4, 1939)

Experiments were made on the orientation of cylindrical particles in viscous flow. Particles were immersed in a moving fluid between two concentric cylinders. Tests showed two different final orientations: (1) a position perpendicular to the plane of the undisturbed flow, and (2) a position parallel to the velocity direction. The relation between the final orientation and the length-diameter ratio of the particle was found for different speeds. Comparison is made between the experimental results and theoretical predictions.

THE motion of particles suspended in viscous flow is a fluid mechanics problem of a somewhat fundamental nature. The motion of such particles is of concern in considerations of the properties of the suspension in bulk. Jeffery[1] and Guth,[2] for example, in their treatments on the hydrodynamical theory of the viscosity of suspensions, have brought out the need for further investigation. It would be desirable to have correlated measurements of viscosity and orientation for a system of suspended particles. As a first step towards that goal, attention was directed toward a study of the orientation of one particle.

The aim of the present study was to obtain experimental data about the orientation of long cylindrical particles to compare with and supplement the theoretical predictions. Preliminary studies indicated two different types of final orientation, depending upon the aspect ratio (ratio of length to diameter) of the immersed particle. Work was done to find the relation between final orientation and aspect ratio.

PREVIOUS WORK

Einstein[3] developed a theory for the case of spheres in viscous flow; his result was deduced strictly from the fundamental equations of hydrodynamics. Jeffery[1] extended Einstein's work to particles of ellipsoidal shape.

Jeffery's mathematical analysis neglected the inertia terms, considered laminar motion with a constant rate of shearing strain, and assumed steady motion outside of the immediate vicinity of the particle. He showed that there is a variety of possible motions corresponding to different initial conditions. The motion is periodic; the axis of the particle is not oriented in any particular direction. The axis of revolution of the particle describes an elliptic cone around the direction of the vortex filaments which is perpendicular to the plane of the undisturbed flow.

Because of the indeterminacy of the motion, Jeffery proposed a "minimum energy hypothesis." One result of this hypothesis is that a prolate spheroidal particle would set itself with its axis perpendicular to the plane of the undisturbed motion, and that the particle would rotate about this axis with constant angular velocity. Jeffery suggested that an experimental investigation of some of his provisional conclusions would be valuable.

Taylor's experiments[4] with aluminum ellipsoids in sodium silicate verified Jeffery's hypothesis. Eirich, Margaretha and Bunzl[5] experimented with natural silk particles in a mixture of tetrachlorethan and olive oil. Because of the importance of this work, it was felt that further experiments would be of value, to explore some features not covered by the previous experiments.

[1] G. B. Jeffery, "The Motion of Ellipsoidal Particles Immersed in a Viscous Fluid," Proc. Roy. Soc. London **102**, 161 (1922).
[2] E. Guth, "On the Hydrodynamical Theory of the Viscosity of Suspensions," Proc. Fifth International Congress for Applied Mechanics (1938), p. 448.
[3] A. Einstein, "Eine neue Bestimmung der Molekül-dimensionen" Ann. der Physik **19**, 289 (1906), with a correction in **34**, 591 (1911).
[4] G. I. Taylor, "The Motion of Ellipsoidal Particles in a Viscous Fluid," Proc. Roy. Soc. London **103**, 58 (1923).
[5] F. Eirich, H. Margaretha and M. Bunzl, "Untersuchungen über die Viskositat von Suspensionen und Losungen, 6. Über die Viskositat von Stabchensuspensionen," Kolloid Zeits. **75**, 20 (1936).

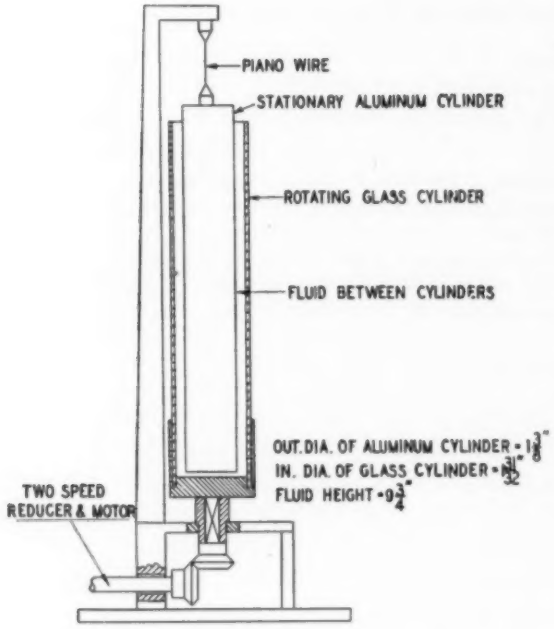
PIANO WIRE
STATIONARY ALUMINUM CYLINDER
ROTATING GLASS CYLINDER
FLUID BETWEEN CYLINDERS
OUT. DIA. OF ALUMINUM CYLINDER = 1 3/8"
IN. DIA. OF GLASS CYLINDER = 1 31/32"
FLUID HEIGHT = 9 3/4"
TWO SPEED REDUCER & MOTOR

FIG. 1. Diagram of apparatus for orientation tests. "Two speed reducer & motor" on left-hand side of figure should read "to speed reducer & motor."

EXPERIMENTAL APPARATUS AND TECHNIQUE

Figure 1 shows the essential features of the apparatus; the two cylinders are concentric. The outer cylinder rotates while the inner is stationary. The space between the cylinders was filled with glycerin. Such an apparatus is convenient, although it does not give exactly the flow assumed in the mathematical treatments. Taylor[4] indicates that it forms a good approximation. Long cylinders were used to reduce end effects.

Human hair from one donor (from a foreign student's head) was used for the particles, and was selected because of its straightness and good rigidity. Particle diameters varied from 0.0028" to 0.0042" while particle lengths varied from 0.018" to 0.090". Only particles of circular section and of uniform diameter were used. Since the surface character is important, a technique of cleaning and gently rolling each particle was developed, to give a smooth surface comparable to that of clean drawn wire.

Each particle was immersed in the fluid midway between the cylinders. The particle was initially set in each of a number of different positions. If a particle kept one definite orientation for eight hours or more, this was considered a "final" orientation.

RESULTS

Table I shows the results obtained with three different speeds of the outer cylinder. The undisturbed flow is in a horizontal plane. All tests were made at 75°F.

"Vertical" means that the particle finally reached a vertical position, within about one or two degrees, and then rotated about the vertical axis of the vortex filament. For example, the particle having an aspect ratio of 10.2, at 4.5 r.p.m., finally became vertical after being initially in each of a number of different positions, including the horizontal. It required from three to 12 hours for a particle to be oriented through 90 degrees from the horizontal. In general, the particles of higher aspect ratio, for vertical orientation, took a longer time than those of lower aspect ratio.

"Horizontal" in Table I means that the particle finally settled in a horizontal plane. For example, the particle having an aspect ratio of 16.5, at 4.5 r.p.m., ultimately reached a horizontal plane after being placed initially at each of various angles with the horizontal. Most of the time the particle was parallel to the velocity direction. At regular intervals the particle would make a short, quick rotation of 180 degrees,

TABLE I. *Orientation of hair particles in glycerin.*

2.7 R.P.M.*		4.5 R.P.M.		7.5 R.P.M.	
ASPECT RATIO	FINAL ORIENTATION	ASPECT RATIO	FINAL ORIENTATION	ASPECT RATIO	FINAL ORIENTATION
32.1		32.1		32.1	
22.9		24.7		23.2	
22.3		22.7		22.7	
21.1	—horizontal	21.1	—horizontal	21.1	—horizontal
20.0		20.0		20.0	
17.2		16.5		17.8	
15.6		15.6		16.7	
				16.5	
14.8		14.8			
14.3		14.4		15.6	
14.0		14.2		15.2	
13.2		12.8		14.8	
12.9		12.2	—vertical	14.1	
12.0		10.2		12.9	
11.8		9.0		12.2	—vertical
10.9	—vertical	7.5		10.5	
10.5		5.1		9.1	
9.8				8.7	
9.2				7.5	
8.8				5.1	
8.5					
7.5					
5.2					

* R.p.m. refers to the speed of the outer glass cylinder.

about a vertical axis, in the horizontal plane. This half-revolution would be completed in about $\frac{1}{3}$ of a revolution of the outer glass cylinder. The time between half-revolutions varied between 1.0 to 1.3 times the time of one revolution of the particle.

Motion Before Final Orientation

The particles approached their final orientations gradually. Before reaching the final positions the particles oscillated in the way indicated by Jeffery's mathematical analysis. The motion was periodic. The motion depended on the initial position of the particle; the axis of the cylindrical particle described an elliptical cone around the direction of the vortex filament perpendicular to the undisturbed flow.

The amplitudes of oscillation were greater in the vertical plane tangent to the cylinders than in the vertical plane through the axis of rotation. Taylor found this on less elongated, ellipsoidal particles.

Motion After Final Orientation

The motion for low aspect ratios, final orientation vertical, agrees with Jeffery's minimum energy hypothesis, while those with horizontal final orientation do not. It has been suggested that particles, whose final orientation is vertical, do not continue the motion given by Jeffery's indeterminate solution because of the neglect of the inertia terms.

The difference in final orientations of particles having aspect ratios from 14 to 16 are not as distinct as Table I might lead one to suspect, because of the long time required for such particles to reach a final orientation.

Various other specimens of human and animal hair and fibers in glycerin were tried, with low and high aspect ratios. The results agreed with those in Table I. Some aluminum cylindrical particles in sodium silicate were tried, to give results agreeing with those in Table I. Sodium silicate, however, was somewhat troublesome; no numerical results are reported because its properties changed with time.

It seems difficult to formulate a simple, rational explanation of this observed difference between final orientations. The answer might lie in a consideration of the terms neglected in the hydrodynamical equations of motion.

Acknowledgment

The author is grateful for the help and advice of Dr. A. A. Potter and Dr. E. Guth. Dr. Potter is Dean of Engineering and Director of the Engineering Experiment Station at Purdue University, while Dr. Guth is connected with the Physics Department of Notre Dame University.

Note on the Behavior of High Intensity Mercury Arcs Falling Freely Under Gravity

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(Received April 8, 1939)

THE experiments of Steenbeck[1] showed that the character of an open air arc in a box was profoundly altered when the effect of gravity in causing convection currents was removed by allowing the ensemble to fall freely under gravity. Thus the arc cross section increased to some 25 times its normal value and its voltage fell to $\frac{1}{3}$ or $\frac{1}{4}$ its normal value.

Suits has recently studied[2] the effect of increasing gravity by centrifuging the arc and has found an increase in gradient which is in accord with his heat transfer theory of the arc.[3]

Recently 400-w type high intensity mercury lamps,[4] operating on a constant a.c. circuit were allowed to fall freely from a height of 64 feet, while a voltmeter connected across the lamp was carefully observed. The voltage dropped in the early part of the 2-sec. interval of flight to a new equilibrium value which though definite, was only of the order of $\frac{2}{3}$ percent below normal (about 1 volt in 150).

This is in general accord with the calculations of Elenbaas[5] that the convection losses in these lamps is relatively negligible, and with calculations of the writer[6] based on Elenbaas' temperature distribution and the actual convection speeds as measured by the method of photographing the motions of fine particles introduced into the arc for the purpose.[6] The present results therefore confirm in a general way Elenbaas picture of these arcs both as regards temperature distribution[5, 7] and streamline flow.[8]

Suits[9] has recently measured the convection current velocity distribution in the open air arc by the method already mentioned and found axial speeds of the order of 3 times those found by the writer in the 400-watt type Hg lamp (130 cm per sec. as compared with 50 cm per sec.). Further he found the convection system to extend for a relatively great distance beyond the margin of the luminous core. Calculations then enabled him to account for 75 percent of the input as being carried away by convection currents. It is therefore understandable that the cessation of these currents following the removal (in effect) of gravity should have such a large effect on the open air arc as Steenbeck found.

That the convection in the Hg lamps is so comparatively small notwithstanding the relatively great density of Hg vapor and gravitational effects thus to be expected is due as Suits[9] has pointed out to the proximity of the tube walls, whose effect is to enhance very greatly the viscous drags on all gas motions, especially in view of the fact that any flow upward in the center must be balanced by a flow downward near the walls!

It was shown by Elenbaas[10] that the convection loss in watts in the high pressure type Hg lamp should be roughly independent of input and of tube diameter and length and should be proportional to the square of the mass (m) of Hg per cm of length of the arc. Since the m values for most other commercial types[6] of Hg arc are smaller than the m value for the 400-watt type[11] it may be concluded that convection losses in these lamps will be still more negligible.

I am indebted to my colleagues, M. A. Townsend, J. A. St. Louis, C. L. Lattin, T. W. Brown, and especially J. D. Forney for assistance in these experiments.

[1] M. Steenbeck, Zeits f. tech. Physik **18**, 593 (1937).
[2] Demonstrated at the third Electronics Conference at Schenectady, Spring 1938.
[3] C. G. Suits and H. Poritsky, Phys. Rev. **52**, 136 (1937); **55**, 1184 (1939).
[4] A 2.9-amp. arc 15.5 cm long operating in unsaturated Hg vapor at about 1.15 atmosphere pressure in a tubular bulb of 3.3 cm internal diameter (see reference 6).
[5] W. Elenbaas, *De Ingenieur* 50 E 83–90 (1935) and other papers in this series.
[6] C. Kenty J. App. Phys. **9**, 53 (1938).
[7] W. Elenbaas, Phys. Rev. **55**, 294 (1939).
[8] W. Elenbaas, Physica **3**, 484 (1936).
[9] C. G. Suits, Phys. Rev. **55**, 198 (1939).
[10] W. Elenbaas, Physica **2**, 169 (1935). See also references 6 and 8.
[11] The m values for the 400-w, 250-w, 85-w, 100-w and quartz 250-w type lamps are, respectively, on the order of 10.5, 3.2, 3, 4 and 10 mg per cm.

Limiting Current Densities in Electron Beams

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In electron beam devices the thermal velocities of the electrons leaving the cathode limit the current density which can be attained with given cathode current density, cathode temperature, potential, and angular spread of arriving electrons. By using Liouville's theorem, limiting expressions are obtained which are valid for any point focus or line focus concentrating system using steady electric and magnetic fields. Current densities in perfectly focused beams are also evaluated. Curves are presented which relate the nearness of approach to the maximum attainable current density with the fraction of cathode current necessarily lost in the concentrating system. Use of the expressions obtained is illustrated by applying them to cathode ray tubes and electron multipliers.

Introduction

IN electron beam devices the thermal velocities of the electrons leaving the cathode limit the current density which can be attained with given cathode current density, cathode temperature, potential, and angular spread of arriving electrons. Several writers have obtained expressions for current density as limited by thermal velocities in special cases.[1–4] D. Langmuir,[5] by applying certain optical laws, obtained an expression for the limiting current density at an image of the cathode or at an exit pupil for an aberrationless axially symmetrical or point focus electron optical system.

It is the purpose of this paper to consider the effect of thermal velocities in limiting the current density in a general way from the physically satisfying viewpoint of statistical mechanics. The expressions obtained by D. Langmuir will be derived, and it will be seen that his expression for limiting current density applies to any electron concentrating scheme involving steady electric and magnetic fields. In addition, similar expressions will be obtained for the current density in the case of essentially two-dimensional, or line-focus, electron optical systems. Some useful curves relating the percentage of cathode current utilized with the nearness of approach to the limiting current density will be presented.

The units used are known as practical units. Potential is measured in volts, current in amperes unless otherwise stated, charge in coulombs, mass in gram sevens, energy in joules, temperature in degrees Kelvin. The magnetic flux density is measured in webers per square cm and the magnetic vector potential in webers per cm. The physical constants utilized have the following values in this system of units:

electronic charge $e=1.59\times10^{-19}$ coulomb
electronic mass $m=9.00\times10^{-35}$ gram seven
Boltzmann's constant $k=1.37\times10^{-23}$ joule per degree.

Expression For Current Density

Just outside of a thermionic cathode it will be assumed that the distribution of electrons is a Maxwell-Boltzmann distribution in which only particles moving away from the cathode are present. For such a distribution the number of particles in a small range of momentum at a given velocity and position is either zero or is given by

$$dN=B\epsilon^{-H/kT}dp_x dp_y dp_z dx dy dz. \qquad (1)$$

Here H is the Hamiltonian energy function, T is the cathode temperature in degrees Kelvin, p_x, p_y, p_z are components of momentum in the x, y and z directions, and B is a constant depending on the density of particles.

[1] I. Langmuir and H. Mott-Smith, Gen. Elec. Rev., 27, July (1924).
[2] H. M. Mott-Smith and I. Langmuir, Phys. Rev. **28**, 727 (1926).
[3] Irving Langmuir and Karl T. Compton, "Electrical Discharge in Gases," Part II, Rev. Mod. Phys. **3**, 220–223 (1937).
[4] R. R. Law, "High Current Electron Gun for Projection Kinescope," Proc. I.R.E. **25**, Aug. (1937).
[5] David B. Langmuir, "Theoretical Limitations of Cathode Ray Tubes," Proc. I.R.E. **25**, Aug. (1937).

The part of the expression

$$B\epsilon^{-H/kT}=\tau \tag{2}$$

is the density of particles in phase space.

For steady electric and magnetic fields

$$H=(1/2m)[(p_x-qA_x)^2+(p_y-qA_y)^2 +(p_z-qA_z)^2]+q\varphi. \tag{3}$$

Here A_x, A_y, A_z are the components of the magnetic vector potential, φ is the electrical potential, q and m are the charge and mass of the particle considered, and p_x, p_y, p_z are the components of the canonical momentum.

Hamilton's equations are

$$dp_x/dt=-\partial H/\partial x, \tag{4}$$

$$dx/dt=\partial H/\partial px, \quad \text{etc.} \tag{5}$$

From (3) and (5) it can easily be seen that in this case the canonical momentum is related to the momentum in the ordinary sense by

$$mv_x=p_x-qA_x, \quad \text{etc.} \tag{6}$$

At a fixed coordinate position x, y, z the vector potential $\bar{A}$ is constant; hence, if ordinary momentum is changed by a certain amount (6) says that the canonical momentum must be changed by an equal amount, or

$$dp_x=d(mv_x), \quad \text{etc.} \tag{7}$$

By using (6) and (7) and taking for the electron the charge q to be equal to $-e$, (1) may be written in a more familiar form. In order to simplify the typography, two dimensionless parameters will be defined, one specifying velocity and the other, potential.

$$u=v(m/2kT)^{\frac{1}{2}},$$

$$\Phi=\varphi e/kT=11{,}600\varphi/T.$$

In the text u will be referred to as velocity and Φ as potential. Eq. (1) then may be written

$$dN=(2mkT)^{\frac{1}{2}}B\epsilon^{-[u_x^2+u_y^2+u_z^2-\Phi]} \times du_x du_y du_z dxdydz. \tag{1a}$$

The equation for particle density was initially written as in (1) because it is desired to use Liouville's theorem to show, subject to certain restrictions, that this expression must give the particle density not only at the cathode but at every point in the beam.

Liouville's theorem states that for a fine dust of particles moving in fields in which the energy is specified by a potential, the particle density in phase space (see (2)) surrounding a given particle remains constant during that particle's motion through the fields. In other words, the total derivative of the density with respect to time, $D\tau/Dt=0$. A derivation is given by Fowler.[6]

Except for the interaction of electrons on a small scale, this is the condition in an electron beam in which focusing is accomplished by steady electric and magnetic fields. Even the gross effect of space charge can be specified by a steady potential; the only portion of the field which is not steady is a very small one due to the discrete nature of the moving charges and their changing positions.

By examining (1) and (1a) it is seen that the particle density in phase space as given by these expressions is constant along the path of any electron, in accordance with Liouville's theorem. This shows that with the same constant B these are correct expressions for the density anywhere in the beam as well as at the cathode surface.

The current density in the x direction as implied by expression (1a) will now be evaluated.

In a small volume element in ordinary space, $dxdydz$, the charge due to electrons having velocities in the range $du_x du_y du_z$ at u_x, u_y, u_z is such that

$$dQ_u=\rho_u dxdydz =\{-e(2mkT)^{\frac{1}{2}}B\epsilon^{-[u_x^2+u_y^2+u_z^2-\Phi]}du_x du_y du_z\} \times dxdydz. \tag{8}$$

The expression in the brackets is, then, the charge density, ρ_u, of such electrons. The current density in the x direction due to such electrons is

$$dj=\rho_u v_x=(2kT/m)^{\frac{1}{2}}\rho_u u_x =-4em(kT)^2Bu_x\epsilon^{-[u_x^2+u_y^2+u_z^2-\Phi]}du_x du_y du_z. \tag{9}$$

This expression can be integrated to give the current density at the cathode surface, j_0. There the allowable values for u_y and u_z range between plus and minus infinity and the limits for u_x are 0 and ∞. Performing this integration, one ob-

[6] R. H. Fowler, *Statistical Mechanics*, second edition, (Cambridge University Press, 1936).

tains

$$j_0 = -2\pi emB(kT)^2. \tag{10}$$

Substituting the value of B given by (10) into (9) and using $u^2 = u_x^2 + u_y^2 + u_z^2$, one obtains

$$dj = (2/\pi) j_0 u_x \epsilon^{-(u^2-\Phi)} du_x du_y du_z. \tag{9a}$$

By integrating (9a) with the proper limits it is possible to obtain a limiting value for the current density for both the point focus and the line focus cases.

Point Focus Case

For the point focus case it will be convenient to use spherical coordinates such that

$$u = u_x^2 + u_y^2 + u_z^2$$
$$u_x = u \cos \alpha$$

and the volume element $2\pi u^2 \sin \alpha d\alpha du$. It will be necessary to integrate with respect to α between the limits $\alpha = 0$ and $\alpha = \theta$, where θ is the maximum angle that electron paths can make with the axis at the point under consideration. All of the electrons arriving at a point in the field having a potential Φ will have velocities equal to or greater than

$$u = \Phi^{\frac{1}{2}},$$

because none can have less than zero velocity of emission from the cathode. The limits of integration with respect to u will then be

$$\Phi^{\frac{1}{2}} \text{ and } \infty.$$

The expression for the limiting current density then becomes

$$j_m = 4 j_0 \int_{\Phi^{\frac{1}{2}}}^{\infty} \int_0^{\theta} u^3 \epsilon^{-(u^2-\Phi)} \sin \alpha \cos \alpha d\alpha du$$
$$= j_0 (1+\Phi) \sin^2 \theta. \tag{11}$$

This is the expression D. Langmuir obtained for the upper limit to the current density. It is seen to be quite independent of the nature of the concentrating system when only steady fields are involved.

This is a limiting expression and does not predict what current density can actually be attained. It is evident that in the case of perfect focusing an electron leaving a point on the cathode with zero initial velocity will arrive at a determined point at the image from a determined direction, and that an electron arriving at the same point at the image from a different direction must have left the point at the cathode with other than zero initial velocity. When, therefore, a limit is placed on θ, the angle of arrival, a limit is implicitly placed on the velocity of emission. The problem in obtaining the actual current is in finding the proper integration limits for velocity corresponding to the allowed value of θ.

An expression for the current density at an image contributed by electrons with velocity u and velocity range du is obtained for the point focus case by integrating the expression leading to (11) with respect to α but not with respect to u, so that

$$dj_u = 2 j_0 \sin^2 \theta u^3 \epsilon^{-(u^2-\Phi)} du. \tag{12}$$

If perfect focusing is assumed, all of the electrons leaving an area $\Delta\tau'$ on the cathode will arrive on an area $\Delta\tau$, regardless of the initial velocities. If the linear magnification is M, then $\Delta\tau = M^2 \Delta\tau'$.

At the cathode where $\Phi = 0$, let the initial velocity be u_0 and suppose θ' is the value of θ. Then the current from $\Delta\tau'$ is

$$di_u' = \Delta\tau' dj_u' = 2\Delta\tau' j_0 \sin^2 \theta' u_0^2 \epsilon^{-u_0^2} u_0 du_0. \tag{13}$$

At the image, for the area $\Delta\tau = M^2 \Delta\tau'$

$$di_u = M^2 \Delta\tau' dj_u = 2M^2 \Delta\tau' j_0 \sin^2 \theta u^2 \epsilon^{-(u^2-\Phi)} u du.$$

But at the plane of the image at potential Φ

$$u^2 = u_0^2 + \Phi, \quad u du = u_0 du_0.$$

Substituting in the relation for current at the image,

$$di_u = 2M^2 \Delta\tau' j_0 \sin^2 \theta u^2 \epsilon^{-u_0^2} u_0 du_0. \tag{14}$$

Since the current di_u' at the cathode from area $\Delta\tau'$ will be the same as di_u at the image over area $\Delta\tau$, it follows from relations (13) and (14) that

$$u_0^2 \sin^2 \theta' = M^2 u^2 \sin^2 \theta. \tag{15}$$

It is convenient to introduce a quantity

$$\beta = M \sin \theta.$$

In terms of this, relation (15) may be rewritten

$$u_0^2 \sin^2 \theta' = u^2 \beta^2. \tag{15a}$$

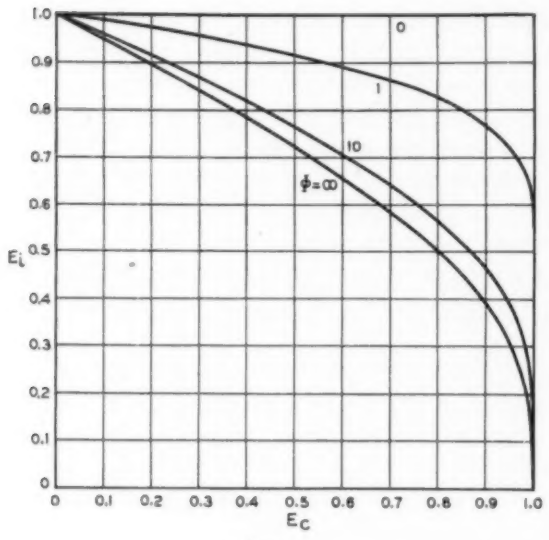

FIG. 1.

If θ' is such that all electrons leaving the cathode are included, its value must be $\pi/2$. Corresponding to this value of θ' there is an angle of arrival θ_u which includes all the electrons leaving the cathode with a velocity u_0. There is a corresponding value of β, β_u.

Substituting for u_0^2 the value $u^2-\Phi$, (15a) gives

$$u=(\Phi/1-\beta_u^2)^{\frac{1}{2}}=f(\beta_u), \tag{16}$$

$$\beta_u=(u^2-\Phi)^{\frac{1}{2}}/u=g(u). \tag{16a}$$

The value of u is then the maximum velocity an electron can have and still reach $\Delta\tau$ along a path making an angle with the axis as large as that defined by β_u. β_u is the maximum value of β for an electron with velocity u.

It can be seen immediately that for values of M less than unity, the sine of the angle defined by β_u may be greater than unity. As this is impossible, only under certain conditions can electrons leaving the cathode at an angle $\pi/2$ arrive at the image, and all of the electrons leaving the cathode can arrive at the image only when M is greater than unity.

Noting that if

$$M\sin\alpha=\gamma$$
$$M\cos\alpha d\alpha=d\gamma,$$

the expression for the current density as used in (11) may be rewritten

$$dj=(4j_0/M^2)u^3\epsilon^{-(u^2-\Phi)}\gamma d\gamma du. \tag{17}$$

It is desired to integrate this expression for all arriving electrons whose paths lie within an arbitrary limiting angle θ, or for all arriving electrons for which γ is less than a value β corresponding to the angle θ.

For velocities lower than $u=f(\beta)$, as obtained from (16), β_u as obtained from (16a) is less than β, and the limiting value of γ is given by $\beta_u=g(u)$.

For higher velocities, β_u is greater than β, and the maximum allowable value of γ is set by the arbitrary limit β corresponding to the limiting angle θ.

Thus the integration is divided into two parts, with limits

$$\gamma=0 \text{ to } \gamma=g(u), \quad u=0 \text{ to } u=f(\beta),$$
$$\gamma=0 \text{ to } \gamma=\beta, \quad u=f(\beta) \text{ to } u=\infty$$

Performing the integration leads to an expression

$$j=(j_0/M^2)[1-(1-\beta^2)\epsilon^{-\beta^2\Phi/(1-\beta^2)}]. \tag{18}$$

For $\beta\geqslant 1$, all the cathode current arrives at the image. For $\beta>1$, (18) does not apply. All the cathode current arrives at the image within an angle θ such that

$$\sin\theta=1/M.$$

As β is made to approach zero, j as given by (18) approaches j_m as given by (11).

It is interesting to note how it becomes necessary to waste an increasing fraction of the cathode current as the limiting current density, j_m, is approached. In considering this matter two quantities will be defined, E_i, the intensity efficiency and E_c, the current efficiency.

The intensity efficiency will be defined as the ratio of the actual current density to the maximum attainable current density, or

$$E_i=j/j_m. \tag{19}$$

If all the cathode current arrived at the image, the current density at the image would be

$$j'=j_0/M^2. \tag{20}$$

The current efficiency or the fraction of cathode current reaching the image, will be

$$E_c=j/j'=M^2j/j_0. \tag{21}$$

At a given value of Φ, E_i and E_c are functions

of β only, and a plot of E_i *vs.* E_c may be made. In Fig. 1 plots of E_i *vs.* E_c are given for several values of Φ. $E_c=1$ for $\beta=1$. This results in a finite value of E_i at $E_c=1$. At $E_c=1$, E_i approaches zero as Φ approaches infinity.

It is seen that for $\Phi>10$ ($\varphi>1$ volt for $T=1{,}160°\text{K}$) the curve for $\Phi\rightarrow\infty$ may be used with little error.

Line Focus Case

In the line-focus case in which the fields are essentially two dimensional, the maximum current density and the current density assuming perfect focusing may be arrived at in a similar manner. In this case it is convenient to use cylindrical coordinates. It is assumed that the line focus is parallel to the z axis. We take

$$u_r=(u_x^2+u_y^2)^{\frac{1}{2}}$$
$$u_x=u_r\cos\alpha.$$

The element of volume is $u_r d\alpha du_r du_z$. It will be necessary to integrate with respect to α between the limits $-\theta$ and $+\theta$, and for values of u_z, the velocity component unaffected by the focusing fields, from $-\infty$ to $+\infty$ to cover all possible values of thermal velocity. Any energy the electron abstracts from the field must appear as a change in u_r. The limits of integration for u_r are accordingly $\Phi^{\frac{1}{2}}$ and ∞. Using these limits, the integration gives for the limiting current density

$$j_m=\frac{2j_0}{\pi}\epsilon^{\Phi}\int_{\Phi^{\frac{1}{2}}}^{\infty}\int_{-\infty}^{+\infty}\int_{-\theta}^{+\theta}u_r^2\epsilon^{-u_r^2}\epsilon^{-u_z^2}\cos\alpha d\alpha du_z du_r$$
$$=j_0\left(\frac{2}{\pi^{\frac{1}{2}}}\Phi^{\frac{1}{2}}+\epsilon^{\Phi}(1-\text{erf }\Phi^{\frac{1}{2}})\right)\sin\theta, \qquad (23)$$

where

$$\text{erf } x=\frac{2}{\pi^{\frac{1}{2}}}\int_0^x\epsilon^{-u^2}du.$$

The second term in (23) may be neglected with an error of less than 10 percent for $\Phi>5$ ($\varphi>0.5$ volt for $T=1160°\text{K}$). The error decreases as Φ increases.

In evaluating the current density for perfect focusing, in the case of the line focus beam, the same procedure may be applied in seeking a lower limit for u_r, and the limit obtained is

$$u_r=(\Phi/1-\beta_u^2)^{\frac{1}{2}}=f(\beta_u), \qquad (25)$$
$$\beta_u=(u_r^2-\Phi)^{\frac{1}{2}}/u_r=g(u_r). \qquad (25a)$$

The current density as used in (23) may be rewritten

$$dj=(2j_0/\pi M)\epsilon^{-(u_r^2+u_z^2-\Phi)}d\gamma du_z du_r. \qquad (26)$$

It is most convenient to integrate for positive values of γ only, and double the result to obtain the current density included in the range $-\beta$ to $+\beta$. The expression will be integrated first with respect to γ, as in the point-focus case. There will be two parts to the integration, with limits

$$u_z=-\infty \text{ to } u_z=\infty, \quad \gamma=0 \text{ to } \gamma=g(u_r), \quad u_r=0 \text{ to } u_r=f(\beta),$$
$$u_z=-\infty \text{ to } u_z=\infty, \quad \gamma=0 \text{ to } \gamma=\beta, \quad u_r=g(\beta) \text{ to } u_r=\infty.$$

Carrying out this integration leads to an expression

$$j=\frac{j_0}{M}\left[\text{erf}\left(\frac{\beta^2\Phi}{1-\beta^2}\right)^{\frac{1}{2}}+\beta\epsilon^{\Phi}\left\{1-\text{erf}\left(\frac{\Phi}{1-\beta^2}\right)^{\frac{1}{2}}\right\}\right]. \qquad (27)$$

For $\beta\geqslant 1$, all the cathode current reaches the image. For $\beta>1$, (27) does not apply and all the cathode current reaches the image within the same angle as for $\beta=1$, so that

$$\sin\theta=1/M.$$

As β approaches zero, j as given by (27) approaches j_m. For large values of Φ the second term becomes negligible with respect to the first.

From these expressions, intensity and current efficiencies can be derived for the line-focus case

$$E_i=j/j_m, \qquad (28)$$
$$E_c=Mj/j_0. \qquad (29)$$

In Fig. 2 plots of E_i *vs.* E_c are given for several values of Φ. $E_c=1$ for $\beta=1$. This results in a finite value of E_i at $E_c=1$. At $E_c=1$, E_i approaches zero as Φ approaches infinity.

It is seen that for $\Phi>10$ ($\varphi>1$ volt for $T=1{,}160°\text{K}$), the curve for $\Phi\rightarrow\infty$ may be used with little error. As in the point-focus case, E_c is always unity for $\beta\geqslant 1$.

Further Considerations

The expressions for j in the point-focus and line-focus cases apply to images of the cathode. It can be shown that these expressions apply also to the current density at an exit pupil—

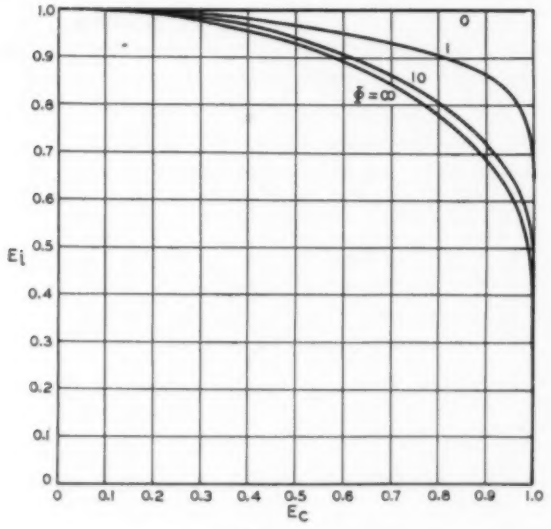

Fig. 2.

that is, a point at which all electrons leaving the cathode with zero initial velocity arrive at the same point. To see this, consider any such minimum cross section of the beam as at A in Fig. 3. Imagine the rays or electron paths extended linearly to form a large image a long distance X_0 away. Imagine that the image has a width W', the exit pupil a width W, the image a maximum angular spread θ' at the axis; the exit pupil a maximum angular spread θ at the axis; the image a current density j' and the exit pupil a current density, j, and both the same current. Extending the diverging rays from the center of the exit pupil to the image, if θ is not large the width of the image is seen to be approximately

$$W' = 2X_0 \sin \theta. \tag{30}$$

Looking back from a point on the axis at the image to the exit pupil it is seen that the allowable angular spread, that of paths coming from the edge of the pupil, is such that, if θ' is not too large,

$$\sin \theta' = (W/2)(1/X_0). \tag{31}$$

If M' is the ratio of image size to cathode size, the ratio of pupil size to cathode size is

$$M = M'W/W'. \tag{32}$$

From (30), (31), (32) it is seen that, within the limits implied by the approximations made

$$M \sin \theta = M' \sin \theta'.$$
$$\beta = \beta'.$$

Thus the terms involving β in (18) or (27) would be the same whether M and θ were measured at the pupil or at the image. It is easily seen, then, that if (18) and (27) applied to the pupil as well as the image, the current densities would be related by the square of the ratio of the magnifications, M and M', in (18) and by the ratio of the magnifications in (27). As this must be so physically, the current in the pupil being the same as the current in the image, (18) and (27) must apply to both pupil and image.

An exit pupil has of itself no definite width, although its width may be limited by the interposition of an aperture or stop. Thus in the case of the exit pupil M refers merely to the ratio the width of the portion of the pupil used or considered bears to the cathode width. In considering the very center region of the pupil, M approaches zero, β approaches zero, and the current density approaches the limiting current density j_m as given by (11) or (23). The density j given by (18) or (27) is the average value over the portion of the pupil used or considered.

Thus in the case of the exit pupil it is easy to see why the current efficiency falls off as the intensity efficiency increases, as is shown in Figs. 1 and 2, for to approach the limiting current density it is necessary to eliminate the current flowing in the outer portions of the pupil by the interposition of a stop, and decrease β in this manner.

In the case of an image of the cathode, the current density is uniform. However, the current density carried by electrons making large angles with the axis is less than the theoretical maximum. Thus in the case of the image of the cathode it is necessary to decrease β by eliminating the more widely diverging electrons in order to bring the current density of the beam close to the limiting value.

To approach the limiting current density, then,

two courses are open: to limit by means of an aperture the current passing an exit pupil, or to eliminate the more widely diverging electrons in a system giving a small image of the cathode.

It may be seen from Figs. 1 and 2 that to obtain a given value of E_i, E_c must be made smaller in the case of a point focus beam than in the case of a line focus beam. This is as might be expected. In the case of a point-focus beam, halving the aperture width to eliminate widely diverging electrons reduces the aperture area to one-quarter. In the case of a line-focus beam, halving the aperture width halves the aperture area. Similar things might be said of the angle of convergence.

In deriving the expressions for actual current density, j, perfect focusing has been assumed, while it is known that aberrationless electron lenses do not exist. For this reason the treatment of a particular focusing system is illuminating.

By use of the principle of conservation of angular momentum it is simple to set up expressions for the anode current density in a given angular range in the cases of concentric spheres and concentric cylinders, with either internal or external anodes. Defining the ratio of anode radius to cathode radius as M, the integrals obtained are exactly those leading to (18) and (27). Thus, defining M in this way, the current density in an exit pupil cut in the anode of such a system will be correctly given by (18) or (27).

Some objection may be made that for (18) and (27) to apply strictly the magnification should relate the pupil width and cathode width, and not the anode radius and cathode radius. This example shows, however, that the assumption of perfect focusing is not unduly optimistic.

At very high voltages, say above 40,000 volts, the change in electronic mass with velocity becomes important and must be taken into account. This may be done by using the equations of special relativity connecting momentum with kinetic energy and velocity. The expression for the limiting current density in the point-focus case is easily integrated to give the relativistic analog of (11),

$$j_m = j_0\left[1+\Phi\left[1+\frac{\Phi}{2(1+mc^2/kT)}\right]\right]\sin^2\theta. \quad (33)$$

L. A. MacColl has integrated the relativistic expression for j_m in the line-focus case in a finite form in terms of special functions. The expression obtained is of some complexity. An attempt to solve for the current density in the case of perfect focusing would result in further complication. As it is felt that the relativistic case is mainly of speculative interest, and that the nonrelativistic expressions are adequate for practical use, this matter will not be further considered here.

Application to Cathode-Ray Tube

It is of interest to consider some applications of the expressions which have been developed. A natural application of the expressions for the point-focus case is in the design of cathode-ray tubes.

A cathode-ray tube, as shown in Fig. 4 generally involves a cathode, say of diameter d_c, whose emission is focused on an aperture of diameter d_1 at a potential φ_1; a field-free space (except for space charge effects, which will here be neglected) to the final lens a distance l_1 away, where the beam has a width W; a change to some potential φ_2 involving a converging action (and generally a change in beam width, which will here be neglected); and another field-free region between the final lens and the spot of diameter d_2 on the screen, a distance l_2 away.

If E_c, the current efficiency, or the portion of the cathode current which may be utilized, is

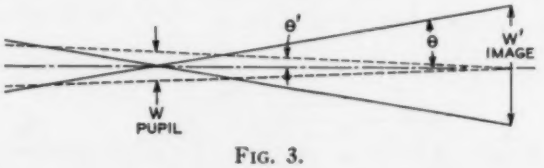

FIG. 3.

fixed, then E_i, the intensity efficiency, or the desired nearness of approach to the limiting current density, can be obtained immediately from Fig. 1. Then if j_0, the allowable cathode current density, and T, the cathode temperature, are known, and the dimensions and potentials shown in Fig. 4 are fixed on, several important operating parameters of the tube can be calculated by the use of relation (11).

In applying (11) to obtain the spot current or beam current one may note that, approxi-

mately, sin $\theta_2 = W/2l_2$. Hence from (11) the beam current i is;

$$i = \frac{\pi d_2^2}{4} E_i j_m = \frac{\pi d_2^2}{4} E_i \left(1 + \frac{11600}{T}\varphi_2\right) \frac{W^2}{4l_2^2}. \quad (34)$$

It is interesting to assume reasonable values and see what the limiting current is. For

$j_0 = 50$ milliamperes/cm^2
$E_c = 0.60$ ($E_i = 0.65$ from Fig. 1)
$T = 1{,}160°$ Kelvin
$d_2 = 0.1$ cm
$W = 0.5$ cm
$l_2 = 40$ cm
$\varphi = 3000$ volts,

(34) gives $i = 0.3$ milliamperes. The cathode current necessary will be given by

$$i_0 = i/E_c = 0.5 \text{ milliampere.}$$

Knowing both the cathode current and the cathode current density, the diameter of the cathode may easily be calculated. It is in this case

$$d_c = 0.113 \text{ cm.}$$

By applying (34) at the first aperture the diameter d_1 necessary to pass the current i can be calculated. Assuming

$l_1 = 5$ cm
$\varphi_1 = 500$ volts,

(34) gives in this case $d_1 = 0.03$ cm. Such computations are easily made and may be very useful in designing beam devices.

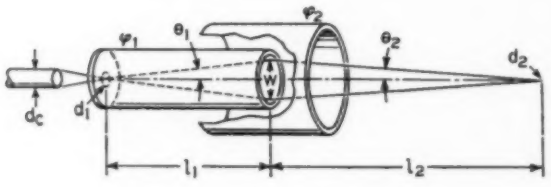

FIG. 4.

APPLICATION TO ELECTRON MULTIPLIER

A natural application of the line-focus results presents itself in the electron multiplier of the essentially two-dimensional type.

Consider an electron multiplier in which the fields between any two successive plates are exactly similar, as will be the case in the structure shown in Figs. 5a and 5b. Imagine that the electrode shapes have been made such that over a considerable portion of the plate surface electrons leaving with zero initial velocity will arrive at a common focus on the next plate as shown in Fig. 5a. The diverging effect of space charge will be neglected. In this case, if the secondary electrons produced had no initial velocities the electron stream passing through the multiplier would have zero width and would arrive at each plate in a sharp line focus, as shown in 5a.

Because of the initial velocities of secondaries leaving the plates, the electron stream will not converge to zero width in passing from stage to stage. If there is an actual converging effect in the multiplier, as has been described, and if no electrons are lost from stage to stage, a condition will be approached after several stages of operation in which the current density of electrons arriving at any region on a multiplier plate is essentially the same as the current density of electrons leaving the corresponding region on the preceding plate. The current density across the width of the beam will not be constant, and the beam will have no definite limits. However, a width W will receive a fraction p of the current leaving the preceding plate. It is desired to estimate this fraction by using the relations derived for the line-focus case.

In applying (27) to secondary emission, it is necessary to give a meaning to $e\varphi/kT$. In thermionic emission $2kT/e$ is the mean energy in electron volts of the electrons leaving the cathode. Thus if, as seems not unreasonable, a Maxwellian distribution is assumed for the secondary electrons, and if the mean energy of the electrons in the distribution is φ_0 volts

$$\Phi = e\varphi/kT = 11600\varphi/T = 2\varphi/\varphi_0. \quad (35)$$

Consider the electron stream after several stages of multiplication, when the equilibrium condition has been reached. Consider the current from a portion of the plate W in width, located symmetrically about the point of focus, going to the corresponding region W wide on the next plate, as shown in Fig. 5b. α is the angle which an electron path connecting the points of focus makes with the second plate, θ is the angle an

electron path leaving the first plate a distance $W/2$ from the focus and arriving at the focus on the second plate makes with a path connecting the points of focus. It is the angle used to obtain β in (27). For small distances, $W/2$, $\sin\theta$ may be considered to be proportional to $W/2$ by a factor k which depends on the shape of the multiplier plates

$$\sin\theta = kW/2. \tag{36}$$

The angle α and the constant k can be obtained from a knowledge of the paths of electrons which start with no initial velocity. These paths can be obtained, for instance, by using a rubber diaphragm analogy of the electric field.[7]

The width of the beam measured perpendicular to the direction of electron motion is for electrons arriving at the plate

$$W' = W\sin\alpha,$$

so that the magnification M in (27) is given by

$$M = \sin\alpha \tag{37}$$

and

$$\beta = (kW/2)\sin\alpha. \tag{38}$$

By using this expression for β, and (35) for Φ, it is possible to calculate the current density j at W' by means of (27). The current efficiency may then be obtained.

Thus for any such region of width W it is possible to calculate the maximum fraction of the current leaving that region which can arrive in a corresponding region on the next plate.

An assumption will now be made which should lead to fairly accurate results. It will be assumed that because of the initial velocities a fraction p of the current starting from any point on one plate will arrive within the region W wide on the next plate. This implies (1) that the section of the plate W wide receives one pth of the total current (2) that the current efficiency of the focusing system between W and W' is p, or $p = E_c$.

If this is so, the quantity E_c is the fraction of the total current striking the plate in a width W, after several stages of operation.

By examining the various relations used, several conclusions may be drawn concerning multiplier design.

[7] J. R. Pierce, "Electron Multiplier Design," Bell Lab. Record **16**, May (1938).

In order to obtain a large current efficiency, β should be large. From (38) it is seen that to make β large, $\sin\alpha$ should be as large as possible, or that the electrons should not arrive at the

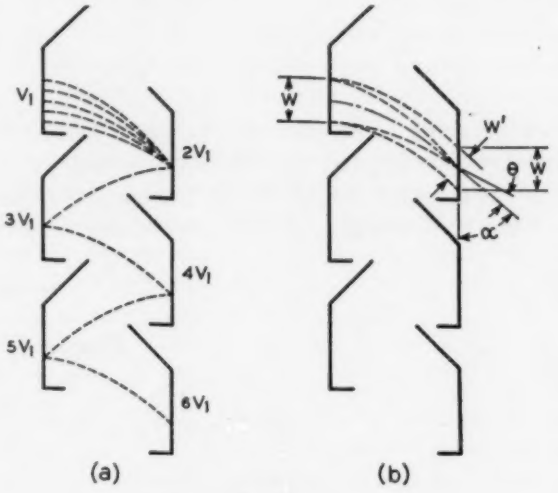

FIG. 5.

plates with grazing incidence. From (38) it can also be seen that k should be as large as possible; that is, that a change in starting position make as large as possible a change in angle of arrival. Because the electrons cannot be brought to a true line focus, it is desirable that the point of focus be near the center rather than near the edge of the multiplier plate.

It is of some interest to compute E_c for certain assumed values of α, k, φ_0, W. If the electrons traveled in straight lines from plate to plate for a distance l, then for small values of $W/2$

$$\sin\theta = (W/2)(\sin\alpha/l)$$

or

$$k = \sin\alpha/l.$$

This is at least as favorable a condition as will be found in actual multipliers. Assuming $l = 1$ cm, $\alpha = \pi/4$,

$$\beta = 0.25W.$$

For secondary electrons, the mean energy φ_0 will be assumed to be of the order of 2.5 volts. Assume φ, the potential change from stage to stage, to be 100 volts, this gives

$$\Phi = 2\varphi/\varphi_0 = 80.$$

E_c, the fraction of current falling within the

width W, has been computed for several values of W:

W	E_c
0.1	0.25
.2	.47
.3	.66
.4	.79
.5	.89

This gives some idea of the most favorable current distribution of electrons falling on a multiplier plate which could be expected, assuming a Maxwellian distribution of secondaries with a mean energy of 2.5 volts. The implication is that in actual multipliers a certain percentage of electrons may be lost by falling outside of the effective plate area.

Acknowledgment

The author wishes to acknowledge gratefully, the contribution of W. Shockley, who first used Liouville's theorem in obtaining the limiting current density, the helpful comments of T. C. Fry, and the interest and assistance of J. O. McNally in the preparation of this paper.

Electrolytic Polishing

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An investigation of the electrolytic polishing of copper by the Jacquet method has thrown new light on the mechanism responsible for the polishing action. From a study of the voltage-current curves it is concluded that when conditions are right for polishing the concentration of dissolved copper at the anode is the highest possible. Accordingly the concentration gradient which exists at the anode limits the rate at which copper can dissolve, i.e., diffuse into the bulk of the electrolyte (orthophosphoric acid). Since there is greater diffusion from raised areas of the anode because of steeper concentration gradients near them, the surface will become progressively more nearly level. The surface does not etch for the concentration gradient alone controls the rate of solution. Details are given for the electrolytic polishing of iron with an orthophosphoric acid electrolyte. A convenient form of cell for electrolytic polishing is described.

METAL specimens are usually prepared for microscopic examination by polishing with a series of graded abrasives. Damage to the surface of the metal arising from this procedure is unimportant for many purposes. In a recent investigation,[1] however, the writer showed that this damage profoundly influences the type of magnetic powder pattern found on iron crystals. It thus appeared desirable to have a convenient polishing technique which would yield undamaged surfaces smooth enough for microscopic examination. The method of electrolytic polishing, devised by Jacquet,[2] was found to be quite satisfactory.

Several observations made while using the electrolytic polishing method have suggested a more detailed mechanism to account for the polishing action than the one proposed by Jacquet. The latter amounts to stating that, as a result of an anode concentration layer, the electrical resistance from the cathode to raised areas on the anode is less than that to the rest of the anode. Hence the anode tends to become smooth since the raised areas are removed most rapidly. It will be here shown that this simple explanation needs to be modified to the extent that diffusion of the dissolved anode material into the electrolyte, and not an Ohm's law resistance, plays the important role. Since some difficulty was experienced in applying the Jacquet method to iron, it seems worth while to report the procedure found to work best for this metal.

Polishing of Copper

The Jacquet method of electrolytic polishing consists of the anodic removal of the specimen surface under conditions which leave the surface

[1] W. C. Elmore, Phys. Rev. **51**, 982 (1937).
[2] P. Jacquet, Comptes rendus **201**, 1473 (1935); **202**, 403 (1936); Bull. Soc. chim. France **3**, 705 (1936).

smooth, rather than etched. For copper and certain copper alloys, Jacquet recommends an electrolyte containing about 530 grams per liter of ortho- or pyrophosphoric acid. When current is first passed through the cell the anode is etched. Polishing occurs as soon as a sufficiently great concentration layer has formed at the anode. Any disturbance to this layer interferes with the polishing; for instance, the anode surface should be horizontal to prevent the layer from flowing away. For best results, the voltage between the anode and a larger copper cathode should be slightly less than the value which will cause the visible evolution of oxygen.

Associated with the polishing of copper is a characteristic behavior of the voltage-current relation for the cell. The curves in Fig. 1 illustrate this behavior. The area of the anode was 1.54 cm². The electrolyte was orthophosphoric acid (sp. gr. about 1.35) which contained enough copper ions to prevent hydrogen evolution at the cathode. If hydrogen is evolved, as in Jacquet's experiments, the curves are shifted to the right by perhaps 0.7 volt. Experiment shows that polishing first occurs when the concentration layer gives rise to a definite back e.m.f., indicated by curve *B*.

Curve *A* in Fig. 1 was obtained before appreciable polarization at the anode had set in. It measures the *IR* drop through the cell. In order to maintain continuously a relatively high current through the cell, for example 40 ma, it was found necessary to increase the applied voltage. When the voltage reached that indicated by curve *B*, the same current could be further maintained only by rapidly increasing the voltage until oxygen evolution occurred, approximately at curve *C*. If the increase in voltage was made somewhat more slowly, a curve such as curve *D* was found. If the cell voltage was held constant from the start, for example at 0.8 volt, the current gradually dropped until curve *B* was again reached. The current-voltage relation then showed the periodic fluctuation which has been reported by Jacquet.[2] The points determining curve *B* were easily obtained by noting where this instability first occurred. That these effects were due to the formation of a concentration layer at the anode was easily verified by agitating the electrolyte near the anode. When further growth of the layer took place but slowly, a curve such as curve *E* was obtained. The limiting current density, however, continued to drop at a very slow rate.

Before attempting an interpretation of the behavior just described it should be pointed out that it is possible to obtain an unlimited number of different voltage-current curves all lying in the region between curves *A* and *C*. For this one need only vary in some arbitrary manner the voltage applied to the cell. The curves shown in Fig. 1 have been selected as having a fundamental significance in regard to the formation of the concentration layer, and in regard to its relation with electrolytic polishing.

It is evident from the curves that the concentration layer plays two distinct roles. In the first place it serves to introduce an opposing e.m.f. in the cell. In the second place, when this back e.m.f. has reached a definite value (as indicated by curve *B*), the layer then serves to limit the current through the cell. The two roles can be better understood by considering the nature of the concentration layer.[3] The layer in the present case is undoubtedly produced by the slow diffusion of newly dissolved anode material into the bulk of the electrolyte. The necessity for the removal of this material by diffusion comes from the fact that the current through the cell is carried, for the most part, by the highly mobile hydrogen ions. If we assume that this explanation is valid and, furthermore, that the law of diffusion holds, then the concentration gradient of dissolved copper at the anode must be

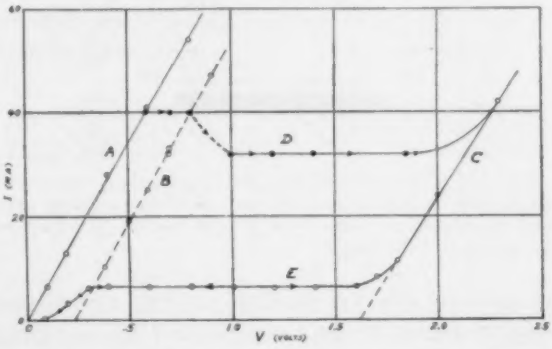

FIG. 1. Voltage-current curves obtained for the polishing of copper (room temperature).

[3] For a recent discussion see J. N. Agar and F. P. Bowden, Proc. Roy. Soc. 169, 206 (1938).

proportional to the current density. The concentration of the dissolved copper there, however, depends in a complicated fashion on the previous history of the cell. An *upper limit* to this concentration is set by the limit of solubility of copper in the orthophosphoric acid electrolyte next to the anode. When the concentration layer has grown to such an extent that this limit is reached, the current density will then be automatically limited by the rate of diffusion of dissolved copper away from the anode, that is, *by the actual concentration gradient then existing at the anode.* Apparently this situation is responsible for curve B, and for the limited current density to the right of this curve. An increase in current density would require a new process to take place such as the precipitation of a copper salt or the liberation of oxygen. Evidently the latter process occurs first, but not until an over-voltage of about 1.6 volts is reached. Thus, along the horizontal portion of curves D or E copper is dissolved at the rate set by the existing concentration gradient. The enhanced back e.m.f. can be attributed to hydrated hydroxyl ions which form a double layer at the anode.[4] When this over-voltage is great enough (*ca.* 1.6 volts) the hydroxyl ions discharge liberating oxygen. The periodic fluctuation in the voltage-current relation occurring at the immediate right of curve B can be explained by assuming that the OH^- ions at the anode decrease the limit of solubility of dissolved copper there. By a simple argument it can be readily shown that this should lead to the sort of periodic fluctuation which is usually observed.

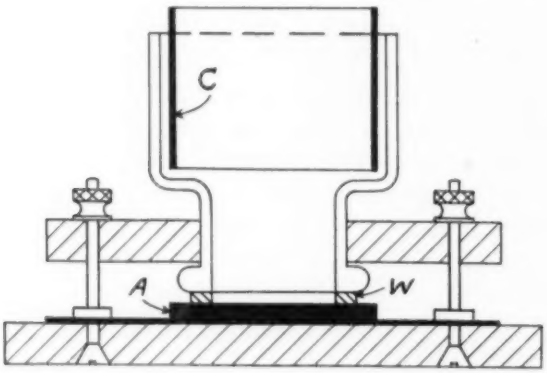

FIG. 2. Cell for electrolytic polishing made from narrow mouth bottle. The cylindrical cathode C has a larger area than the specimen anode A. A rubber washer W separates the bottle and the anode.

The polishing effect (which takes place to the right of curve B) occurs when the rate of solution of copper is limited by the concentration gradient existing at the anode. Since this gradient is greatest near projecting portions of the surface, these portions will dissolve more rapidly than the rest of the surface. The surface, accordingly, will be made more nearly level. For instance, consider the case of a small hemispherical boss in an otherwise plane anode. The surfaces of equal concentration of dissolved copper are the same as the equipotential surfaces for the electrostatic case,[5] since the diffusion equation, $\partial c/\partial t = D\nabla^2 c \doteqdot 0$ for the very nearly steady state. Hence, by referring to the electrostatic case, we see that the copper will be removed from the tip of the boss three times as fast as from the plane at distant points, whereas no copper will be removed at the junction of the hemisphere and plane. The general effect is thus to remove the boss. The surface is not etched since the concentration layer in the electrolyte alone controls the rate at which the anode dissolves. Etching can only occur where part of this control is furnished by the anode.

POLISHING OF IRON

Iron or silicon-iron specimens consisting of a single phase can be polished by an orthophosphoric acid electrolyte[6] provided conditions be carefully controlled.[7] It is found that the current density at the anode must be quite uniform and that the cell voltage must be kept within a narrow range which depends on the current. Furthermore, the electrolyte should contain some dissolved iron so that the anode is not too readily etched by the acid in the absence of any current through the cell.

[4] This is the view taken in the Gurney theory of over-voltage. R. W. Gurney, Proc. Roy. Soc. **134**, 137 (1931).

[5] See, for example, L. Page and N. I. Adams, *Principles of Electricity* (D. Van Nostrand Company, New York, 1931), p. 93.

[6] In a recent note (Comptes rendus **208**, 1012 (1939)) P. Jacquet and P. Rocquet describe the polishing of iron by an electrolyte containing a mixture of concentrated acetic and perchloric acids.

[7] This was mentioned in reference 1. The electrolytic polishing of cobalt has also been reported: W. C. Elmore, Phys. Rev. **53**, 757 (1938). Cobalt behaves similarly to iron except that curves B and P in Fig. 3 are more widely separated.

To secure the necessary uniform conditions at the anode, the area to be polished can be made the bottom of the cell. Fig. 2 illustrates such a cell made from the upper portion of a narrow mouth bottle. This arrangement insures uniform current density at the anode and prevents the concentration layer from flowing away. The anode can be viewed from above and can be conveniently swabbed if any gas bubbles adhere to it. In addition, this form of cell is useful for polishing areas on large specimens.

Typical voltage-current curves obtained for an electrolyte of sp. gr. 1.316 are shown in Fig. 3. The anode consisted of Armco iron. The area polished was 3.8 cm². The straight portion of curve *A*, taken before polarization set in, does not extend to the origin since hydrogen was evolved at the cathode. Curve *B*, having the same significance as in Fig. 1, was found to be somewhat more difficult to determine, for the periodic instability characteristic of the copper cell was less pronounced. The points shown were taken by observing where the voltage applied to the cell had to be increased abruptly in order to maintain the current through the cell at a chosen value. A new feature not found with copper is curve *P* where the anode became passive. It is to be noted that passivity set in at a definite back e.m.f., not at a particular current density. To the right of curve *P* the current dropped to a low value (curve *F*). At approximately 2.2 volts the curve began to rise with oxygen being liberated. The transition to the passive state could be reversed by lowering sufficiently the cell voltage. Polishing was found to occur in the region between curves *B* and *F*, where curve *E* is typical.

The explanation proposed to account for the polishing of copper seems also to hold for iron. The passivity which occurs with iron simply

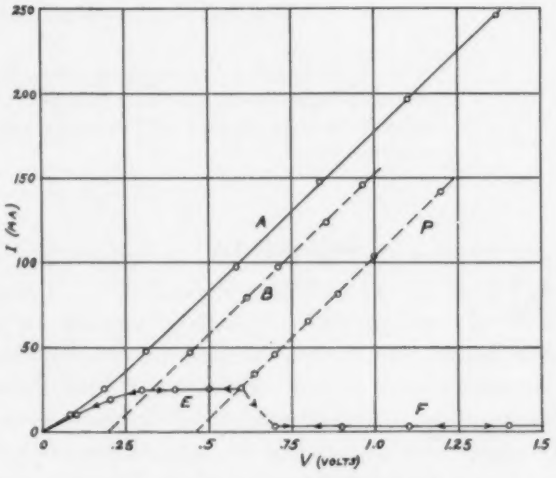

FIG. 3. Voltage-current curves obtained for the polishing of iron (room temperature).

means that hydroxyl ions can discharge at a particular back e.m.f. to form a passive layer on the anode. Before this occurs the solution of iron is limited, as with copper, by the concentration gradient existing at the anode.

Specimens of iron polished electrolytically are quite noticeably brighter in appearance than those polished mechanically by the usual method. Examination under high power magnification shows no structure within a single crystal grain. The electrolytic method is not suitable for preparing mirrors of macroscopic dimensions since the surface tends to become slightly wavy. This macroscopic unevenness does not interfere with microscopic examination at medium or high magnification.

The Temperature of High Pressure Arcs

C. G. SUITS
General Electric Company, Schenectady, New York
(Received April 17, 1939)

From measured values of arc gradient E (v cm^{-1}) and current density I (amp. cm^{-2}) the arc temperature is calculated for nitrogen in the pressure range 1–30 atmospheres and current range 1–10 amperes, and in hydrogen at one atmosphere pressure for the current range 1–10 amperes.

INTRODUCTION

NONE of the available means of measuring arc temperatures has been applied with satisfactory accuracy in the pressure range above one atmosphere, where experimental difficulties are such as to discourage direct measurement. Gradient and current density measurements are available, however, for nitrogen in this pressure range, and it is of interest to calculate the arc temperatures from these data. For this purpose we use a simple form of the ionization equation, neglecting refinements to this relation as not being warranted in this application.

ARC TEMPERATURES FROM E AND I

The ionization equation has the following convenient form

$$\log_{10}\left(\frac{n_0^2}{N}\right)=-\frac{5040\,V_e}{T}+1.5\log_{10}T+15.38, \quad (1)$$

where n_0 is the electron density in the region of constant gradient, N is the molecular density, V_e the ionization potential, and T the absolute temperature. The available evidence indicates that in the arc in air at atmospheric pressure a thermal equilibrium is established[1, 2, 2a] in about 0.001 second.[3] From studies of arc temperatures by the sound velocity method[4] it is known that within the errors of measurement and the uncertainties in calculating the gas constants at high temperature (1) is a quantitative relationship by which T can be calculated from n_0, N, and V_e. In general V_e is unknown, but measurements for a variety of electrode materials in arcs in air show that for tungsten, pure carbon, and polished copper, at low currents, V_e is given satisfactorily by the ionization potential of the gas and is not appreciably lowered by a vapor pressure of metal or carbon. Thus, for many arcs in air V_e has the value of approximately 15 volts. This knowledge must be applied with caution to other experimental conditions, as, for example, other gases at the same or higher pressures, but it is the only method at present for determining T in many of these cases. From $I=n_0e\mu E$, where e is the electronic charge, μ the mobility, I the current density, n_0 the electron density, and E the electric gradient, we obtain

$$(n_0^2/N)=Ap(I/E)^2, \quad (2)$$

where A is a constant involving the Loschmidt number and the mobility constant for each gas, and p is the pressure. Thus (n_c^2/N) can be calculated from I, E and p. If the experimental conditions allow a knowledge of V_e, T can then be calculated. For this purpose it is convenient to plot the right-hand member of (1) with V_e as a parameter, as in Fig. 1. Values of A are given in Table I. Mobility has been calculated from the Langevin equation, using kinetic theory values for the mean free path.

When I is calculated from $i/\pi R^2$, where $2R$ is the photographically determined diameter of the

TABLE I.

	A ($p=1$)
Mercury vapor	20.6×10^{10}
Argon	1.87
Nitrogen	2.73
Helium	0.23
Hydrogen	0.76

[1] L. S. Ornstein, Physik. Zeits. **32**, 517 (1931).
[2] L. S. Ornstein and H. Brinkman, Proc. Roy. Acad. Amsterdam **34**, 33 (1931).
[2a] E. Olsson, Arkiv f. Mat., Astron. och Fysik **26B**, 1 (1938).
[3] H. Witte, Zeits. f. Physik. **88**, 415 (1934).
[4] C. G. Suits, Physics **6**, 315 (1935).

arc column in the region of constant gradient, it is assumed that I is a constant across the section of the current conducting path. This cannot be strictly true, since there must be a radial temperature and electron density gradient. The quantity determined therefore is an effective temperature which, were it constant across the section of the arc, would provide the proper constant electron density for the observed electric gradient. There is much experimental evidence which indicates that in arcs in dissociating gases the radial temperature gradients are small. Supporting this evidence is the fact that the current density I determined from photographs of arcs in air, nitrogen, and hydrogen is nearly constant over a great range of current.

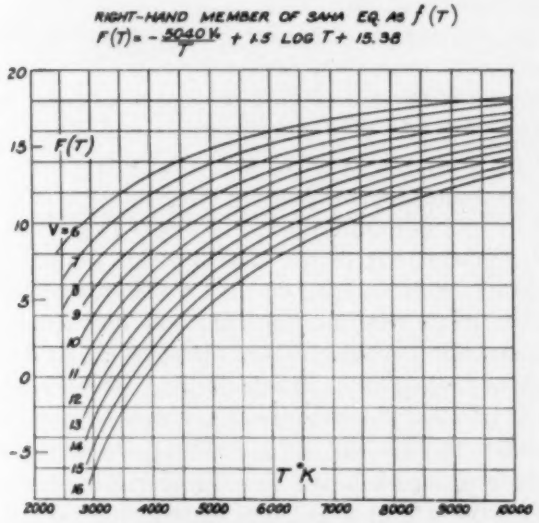

FIG. 1.

In monatomic gases the current density measurements are less conclusive, but from the absence of a sudden change in thermal conductivity at high temperature, as, for example, due to dissociation in diatomic gases, it can be inferred that these arcs have larger temperature drops across the core. A current density calculated from a photographic diameter is in this case a much less definite quantity.

TABLE II. N_2 *arc temperature* (T°K) *for* $V_c=15$.

	i (AMP.)	1	2	5	10
$p=$ 1 atmos.	T°K	5950	6020	6250	6400
$p=$ 5		6400	6450	6770	7050
$p=$ 10		6680	6780	7160	7470
$p=$ 20		7030	7150	7620	7980
$p=$ 30		7230	7400	7920	8320
$p=$ 100					8800*
$p=$ 1000					10200*

* Extrapolated values.

We proceed to determine the arc temperature T under some of the experimental conditions where direct measurement would be extremely difficult, if at all possible. From previously published curves[5] and $I-p$ data[6] for nitrogen we obtain T for the nitrogen arc in the current range 1–10 amperes and in the pressure range 1–30 atmospheres, as shown in Table II.

The temperature at 100 atmospheres and 1000 atmospheres are extrapolations from $\log T-\log p$ plots. T increases both with current and pressure, so that the highest arc temperatures are reached at the highest possible current and pressure. It appears that steady-state gas temperatures as high as 10,000°K are reached in the nitrogen arc in the 1000 atmosphere pressure range.[7]

From the hydrogen E[8] and I[6] data previously published we obtain the hydrogen arc temperatures of Table III. The hydrogen arc temperatures are appreciably higher than found in the nitrogen arc, being at one atmosphere comparable to nitrogen at 10 atmospheres.

TABLE III. *Hydrogen arc temperatures* ($V_c=14$).

	i (AMP.)	1	2	5	10
$p=1$ atmos.	T°K	6500	6600	7000	7400

[5] C. G. Suits, Phys. Rev. **55**, 561 (1939).
[6] C. G. Suits, following paper.
[7] C. G. Suits, J. App. Phys. **10**, 203 (1939).
[8] C. G. Suits, J. App. Phys. **10**, 652 (1939).

Current Densities, Lumen Efficiency and Brightness in A, N_2, He and H_2 Arcs

C. G. Suits
General Electric Company, Schenectady, New York
(Received April 19, 1939)

From oscillographic records of arc diameter D, gradient E, current i, and comparative brightness data, the lumen efficiency and current densities are determined for the positive column of arcs in A, N_2, He and H_2 in the pressure range between 1 and 100 atmospheres.

Introduction

BY the oscillographic method recently described[1] for measurements on high pressure arcs the photographic image of the arc diameter is recorded together with arc voltage e and gradient E (v/cm) as a function of current for a given experimental condition. The current density, lumen efficiency, and brightness data obtained during this study of arcs in A, N_2, He and H_2 are reported in the present paper.

Current Densities

As described in the above reference,[1] the photographic diameter of the arc column is recorded on the oscillogram as a function of arc current. The accuracy of the method is determined chiefly by the degree to which the discharge column has a well-defined luminous boundary. It is an experimental fact that the column of the high pressure arc, particularly in dissociating gases, is better defined than in the low pressure arcs, especially in mercury vapor. That this is so appears from a consideration of the radial temperature gradients in the arc. We can estimate these gradients from arc data and the Fourier heat equation,

$$d\theta/dt = -kAdT/dr. \qquad (1)$$

At the luminous boundary the temperature gradient dT/dr must be sufficient to account for the total electrical energy, and can be calculated from this total energy if the temperature and the thermal conductivity k are known. In Table I this calculation has been made for experimental values of per unit length arc energy for arc temperatures, which are directly measured values in air, and calculated values in helium and mercury. Thermal conductivity k has been calculated for an average between the temperature of the center of the arc core and room temperatures from the extrapolated viscosity data of Hassé and Cook,[2] neglecting the dissociation contribution in the case of air. It can be seen that the thermal gradients are very high, particularly in air.

In contrast to these large temperature gradients in the boundary region the actual measurements (reference 1 of Table I) of the temperature drop across the photographically recorded section of the *cores* of arcs in air show that this quantity is less than the experimental uncertainty of 300°. More recent unpublished results of an exploration of temperatures in the nonluminous boundary regions of an air arc by the *schlieren* method[3] have shown that the temperatures depart very little from ambient values up to within a few millimeters of the luminous region. For a given flow of heat the temperature gradient will be smaller the greater the thermal conductivity of the medium from the Fourier equation above. Since there is an abrupt increase in the thermal conductivity of air at a temperature of 3000° where dissociation becomes appreciable,[4] the small temperature gradients observed across the arc core must therefore be ascribed to the great increase in heat transfer due to the dissociation effect. Hence, the temperature drop across the arc core tends to be very small, the temperature drop in the boundary very large, in arcs in dissociating gases. It is recognized, of course, that there is no discontinuity in conductivity, luminosity, or temperature at the arc "boundary," and hence no real boundary in a strict sense. However, the large temperature gradients

[1] C. G. Suits, Phys. Rev. **55**, 561 (1939).

[2] H. R. Hassé and W. R. Cook, Proc. Roy. Soc. **A125**, 196 (1929).

[3] Although these results have not been published, the method is described in Gen. Elec. Rev. **39**, 430 (1936).

[4] C. G. Suits and H. Poritsky, Physics **6**, 190 (1935).

in the arc boundary region in dissociating gases lead to a sudden decrease in light intensity as well as electron density, which for the present purpose will serve to bound the discharge column. Since the luminosity I may be expected to vary with temperature T as $e^{-E_0/KT}$, a decrease in T of 2000° (from 6000°K) will decrease I by a factor of 11 times at 5000A. That this region of rapid decrease in light intensity cannot be far removed from the boundary for current conduction is evident from the fact that in the same temperature interval the electron density n_0 will decrease by a factor of 2×10^3. Since the current density $I=n_0e\mu E$ (e=electric charge; μ=mobility; E=electric gradient) is proportional to the electron density n_0, I varies with temperature in a manner determined by the

TABLE I. *Temperature gradients at luminous arc boundary.*

Arc in air[1]	6200°K.....	4700 deg. per mm
Arc in He[2]	5600°K.....	1450 deg. per mm
Arc in Hg[3]	6500°K.....	600 deg. per mm

[1] Temperature measured by sound velocity, Physics, **6**, 315 (1935).
[2] Temperature calculated from E and I (J. App. Phys. **10**, 203 (1939)).
[3] W. Elenbaas, Physica **1**, 211 (1933).

Saha equation, if equilibrium conditions exist at all points. (There is an additional small effect due to the temperature dependence of mobility μ, which can be neglected in the present consideration.) That the edge of the luminous core of arcs in air coincides with the boundary of the conducting column can be demonstrated very simply with probes. As a probe approaches the arc column, no appreciable current flows in the probe circuit until the luminous core is reached, where an abrupt increase in current to the probe takes place.[5]

The possibility of an ultraviolet boundary that differs appreciably from the visible boundary was explored in the case of the argon arc by projecting the arc image through a quartz system on a fluorescent screen. The fluorescent image appeared to coincide with the visible image and was in any event not larger than the visible image.

If the arc column were not well defined, large photographic errors might be expected from variations of exposure within a given experiment, as, for example, those due to the variation of light intensity with current. The variation of light intensity with current in the oscillographic records, however, is of the order of three times. When a variation of 30 times in exposure is used in a given set of records, the change in the apparent photographic diameter is well within the spread of the measurements. It can be seen from the considerations just given that in the high pressure arc, especially in dissociating gases, (1) the luminous boundary tends to be sharp, (2) the conductivity decreases sharply with temperature, and (3) the conducting boundary and the luminous boundary will for that reason not differ greatly. A detailed investigation of this question by the method Fischer and König[6] used with the mercury arc would be desirable.

The direct photographic method of determining the arc diameter appears to be the only one available for an experiment of this kind in which a great range of conditions must be studied. The method has yielded satisfactory results in the case of N_2 and H_2. In the case of the arcs in helium and argon the definition of the arc boundary is such that the measurement is made with difficulty even at one atmosphere pressure. At higher pressures the columns of these arcs, especially in argon, develop a double boundary structure for which the present method of measurement is inadequate, and an interpretation is lacking.

By dividing the current by the area, we obtain a quantity I, which may be interpreted as an average current density across the core of the arc. The data on I for nitrogen in the 1–30-atmosphere pressure range, and for hydrogen, helium

TABLE II. *Current densities in* N_2.

	i (AMP.)	1	2	5	10
p= 1 (atmos.)	I (amp. cm^{-2})	6.3	6.1	5.8	5.7
p= 5	I	11.8	9.9	11.6	13.3
p=10	I	18.9	16.7	20.2	24.5
p=30	I	60	48	63	79

TABLE III. *Current densities in* H_2.

	i (AMP.)	2	5	8	10
p=1 (atmos.)	I (amp. cm^{-2})	590	550	540	570

[5] E. H. Bramhall, Proc. Camb. Phil. Soc. **27**, 421 (1931).

[6] E. Fischer and H. König, Physik. Zeits. **39**, 313 (1938).

TABLE IV. *Current densities in* He. (*Pure carbon electrodes.*)

	i (AMP.)	1	2	3	4	5	6	10
$p=1$ (atmos.)	I (amp. cm^{-2})	46	45	48	48	48	46	46

TABLE V. *Current densities in argon.* (*Pure carbon electrodes.*)

	i (AMP.)	1	2	3	4	6	10
$p=1$ (atmos.)	I (amp. cm^{-2})	42	36	33	31	27	25

and argon at a pressure of one atmosphere are given in the Tables II to V.

If we consider first the nitrogen data, it can be seen that the variation of I with i is a small effect, even at the higher pressures. The variation of I with pressure is larger, however. When the log of the arc diameter D is plotted with the log of p as abscissa, the curve is concave toward the axis of pressure. As an approximation

$$D = \text{const. } p^{-\gamma}$$

for nitrogen. The values of γ for the pressure range 1–30 atmospheres are given in Table VI.

The current densities for hydrogen are larger than nitrogen by about two orders of magnitude. The dependence upon current is a small effect in this range of observation.

The data for the argon and helium arc, although subject to a good deal of experimental uncertainty, are presented as the best available. Because of the structure of the discharge and the difficulty in interpreting the records no values are given for pressures above one atmosphere. The current densities in argon show a marked dependence on current, an exception in this respect.

LUMINOUS EFFICIENCY AND SURFACE BRIGHTNESS OF ARCS IN N_2, H_2, He AND A

In this series of measurements tungsten electrodes were used in arcs in the following pressure ranges: H_2 (1 atmos.), N_2 (1–100 atmos.), He (1–50 atmos.), and A (1–100 atmos.). The oscillograph lens aperture and filter system were adjusted to give a "standard" density on the film for a controlled development exposure of a photometered 10-atmosphere Hg capillary arc. Arcs in the above gases were then recorded, with the exposure adjusted to give the standard density on the film for all pressures. Surface brightness of the arc column considered as a solid cylinder was then compared to the Hg capillary from the known filter factors and aperture differences. An approximate value for the lumens per watt in the positive column was determined from the brightness ratios, the dimensions, and the wattage per cm of arc column, —the last calculated from the observed gradient and current.

These observations are recorded in Table VII for an arc current of five amperes.

TABLE VI. γ *for nitrogen arc.*

i (AMP.)	γ
1	0.30
2	0.30
5	0.35
10	0.38

TABLE VII. *Lumens watt*$^{-1}$ *and lumens cm*$^{-2}$ *for* N_2, H_2, He *and* A.

GAS	p (ATMOS.)	L/w	L/CM^2
N_2	1	2.3	280
	10	3.1	1120
	50	17.	11500
	100	27.	36500
H_2	1	0.00024	110
He	1	0.4	112
	10	0.85	308
	50	1.7	1540
A	1	0.4	70
	10	3.0	1100
	50	10.7	6400
	100	17.	22000

The luminous efficiency and the brightness both vary with gas pressure in an unpredictable manner. The pressure change of one to 100 atmospheres increases the brightness of the nitrogen arc by a factor of 127 times and the argon arc by 315 times. Although the lumens per watt at $p=1$ atmos. is negligibly small for these arcs, considered as light sources, this quantity reaches an appreciable magnitude at 100 atmospheres. At one atmosphere pressure the surface brightness of the arcs in H_2, He and A is about the same, while the N_2 arc is about three times brighter. The brightness of the 10-atmosphere Hg lamp is about 10 times greater than the one atmosphere nitrogen arc.

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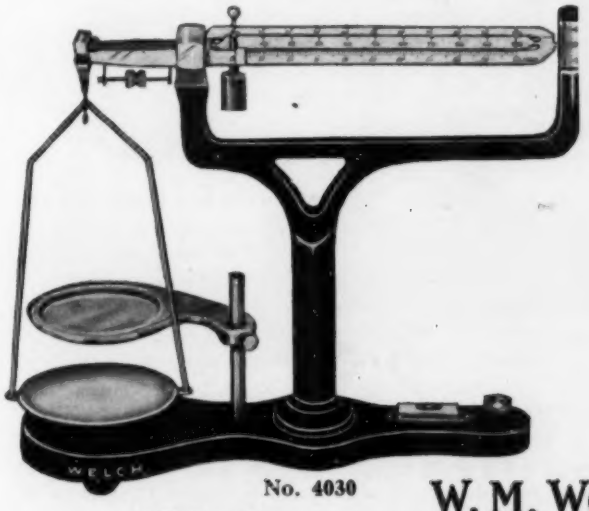

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James G. Biddle Company, 1211–13 Arch Street, Philadelphia, Pennsylvania, announces a new Jagabi rheostat known as "Lubri-tact." The "Lubri-tact" construction consists of a small graphited carbon brush mounted in such a way that it slides along the winding with the laminated phosphor-bronze brushes and provides just enough lubricant to prevent abrasion, even though the temperature of the winding may be high. The phosphor-bronze brushes carry the current; the graphited carbon brushes provide the lubrication.

RCA Sound Catalog

A 56-page catalog containing a complete listing of all RCA sound equipment for a wide variety of applications in the industrial, entertainment and educational fields, has been announced by the Commercial Sound Section of the RCA Manufacturing Company. Special sections are devoted to electric chime systems, recording and instantaneous playback instruments, complete sound systems with single master control wired and wireless intercommunication systems and a complete line of microphones, amplifiers, loudspeakers, standard and multi-wave Antenaplex systems, record players, and hearing aid devices.

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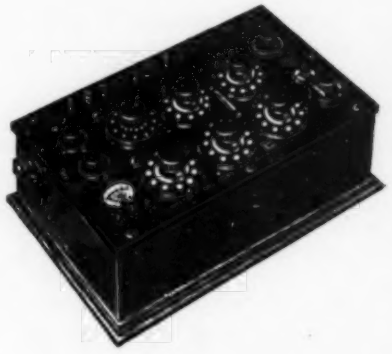
During the past several years laboratory practice has had increasing need for safe and convenient control of gas deliveries at higher pressures than normally supplied by the ordinary laboratory regulator. A series of regulators for gas deliveries at high pressure has recently been developed by Hoke, Incorporated, 122 Fifth Avenue, New York, New York. These new regulators, known as Series 500, have features which will appeal to laboratory workers in this particular field.

Two standard types are available, one for delivery pressure up to 550 lb. per sq. inch and the other for pressures up to approximately 1000 lb. per sq. inch. All regulators, while originally designed particularly for dead-end service, are conservatively rated for flows as high as 50 cu. ft./hr. They are available for use with all non-corrosive gases, and the gas to be used should always be specified.

Thermometer Bridge

Accurate measurements of temperatures from those in the neighborhood of absolute zero up to about 500°C are possible on the new Mueller resistance thermometer bridge. Measuring from 0 to 111.111 ohms in steps of 0.0001 ohm, it establishes a new level of accuracy for commercially available temperature measuring equipment in this range. Achieving this high accuracy by virtue of a constant-temperature chamber for the measuring resistors, the new bridge also has many other distinctive features fully described in Catalog E-33C(1) which will be sent upon request. Write to Leeds and Northrup Company, 4934 Stenton Avenue, Philadelphia, Pennsylvania.



Electro-Chemograph

Applying the dropping mercury electrode principle of analysis, Leeds and Northrup's new Electro-Chemograph automatically inks curves from which quantitative and qualitative analyses can be determined. Making possible quick and convenient analyses in those processes where its possibilities have been determined, the method eliminates dark rooms, developing processes and delicate galvanometer systems. Drawn in sight of the operator the curve is inked on a wide, easily-read, permanent chart. This equipment is described in L and N's Technical Publication E-94(1). A copy will be sent upon request to Leeds and Northrup Company, 4934 Stenton Avenue, Philadelphia, Pennsylvania.

Recent Booklets Received

Tremendous Trifles a new booklet issued by the International Nickel Company, Incorporated, 67 Wall Street, New York. This new booklet contains 16 pages and presents technical and mechanical information on nickel, monel, iconel, "Z" nickel, "K" monel, and other high nickel alloys as used for such parts under service conditions where ordinary materials might fail.

★

The July, 1939, issue of the *General Radio Experimenter* published by General Radio Company, Cambridge, Massachusetts, describes and illustrates the two new models of the Variac making the amount of power controllable by a single unit more than double. Type 50-A, for 115-volt service, will handle 5 kva, and Type 50-B, operating at 230

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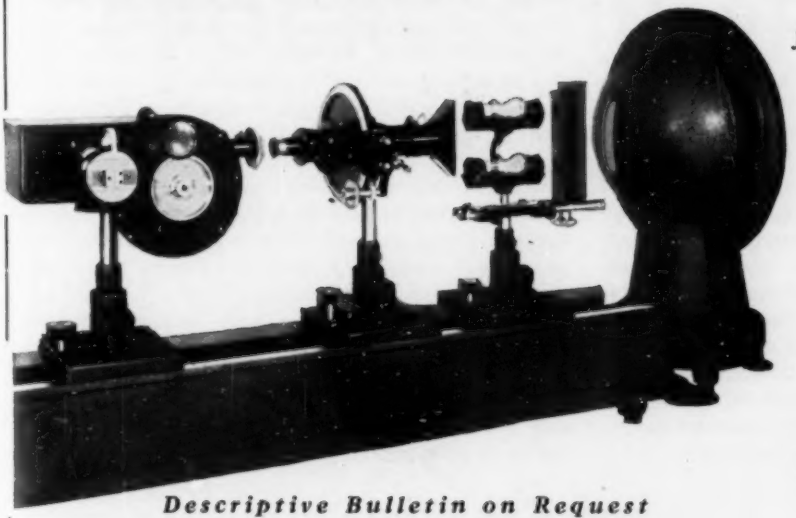

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Although introduced two years ago, the 544-B megohm bridge was not used for the higher voltage because of the bulkiness of the necessary batteries. The *General Radio Experimenter* in the June, 1939, issue describes the new 544-P3 500-volt power supply which allows the full possibilities of the bridge to be realized.

★

Dumont Oscillographer, Nos. 4 and 5, June and July, 1939. This is a four-page pamphlet dealing with the induction tests of ignition coils with cathode-ray oscillograph.

★

Roller-Smith Company Catalog No. 123, July, 1939. This twenty-page catalog discusses portable instruments for the direct and alternating current. One of the new instruments described is a wattmeter designed for measuring the power flow in household appliances. It has two ranges, one of 0–250 watts, and the other, 0–750 watts.

★

Roller-Smith Company Catalog 12A, July, 1939. This 8-page catalog describes a new line of thermo-magnetic relays. These relays are designed for the protection of alternating-current apparatus which is subject to overloads, either momentary or continuous. The time delay element is a new type of device consisting of a heater and bi-metallic strip combination, fully compensated for ambient temperature. Thus the relay can be located anywhere remote from the apparatus being protected and always operate on current values directly without influence from any other variables. The heating rate of the bi-metallic strip is controlled so as to match approximately the heating rate of windings of the protected apparatus. The instantaneous element operates at 10 times the minimum operating current which is determined by the tap setting.

Necrology

Sir George Kendrick, who was actively associated with the development of the University of Birmingham, where he endowed the Poynting chair of physics, died on May 28, at the age of eighty-nine.

Electrical Wave Shown in Slow Motion

An apparatus to illustrate in slow motion the movement of electrical waves is being displayed by the Westinghouse Electric & Manufacturing Company. C. F. Wagner, electrical engineer in the company's central station division, is the inventor.

By substituting metal arms, springs and damping or resistance elements for their electrical counterparts, Mr. Wagner has produced a model of a transmission line on which he can dramatize a surge of current and prolong its life from one ten-thousandth of a second to as much as five or even ten seconds. That is long enough for the engineers to study visually what happens when lightning strikes a transmission line or when a short circuit occurs on the line.

Resembling a streamlined xylophone, the wave model device consists of 56 long narrow aluminum arms mounted at their center of gravity on hardened steel bearings. A flat spring is mounted rigidly to each of the arms and the free end of the spring is attached to an adjacent arm. When the first arm is moved up or down it transmits the movement successively to all the other arms in the form of a wave, just as a long rope can be made to move in waves when it is snapped at one end.

As long as all 56 arms are allowed to move freely, a wave started at one end will move forward to the other end and return, always staying above the imaginary line formed by the arms at rest. Mr. Wagner has explained that this corresponds to positive electrical waves moving in an open circuit. However, if the fifty-sixth arm is prevented from moving, in simulation of a short circuit, the returning wave is negative, moving below the imaginary line.

Mr. Wagner can even reproduce the effects of a lightning arrester on his wave model, showing how the arrester acts like a dam to hold back the abnormal voltage of a lightning stroke and permit only a safe amount of voltage to continue along the transmission line to the electrical apparatus which it is protecting. When a V-shaped metal bar is moved into place above one of the aluminum arms it limits the rise of the arm. "Thus," the engineer has explained, "if the arrester location is shown near the middle of the model and a wave is produced on the left-hand side, only a wave of constant amplitude equal to the arrester voltage is permitted to pass the arrester."

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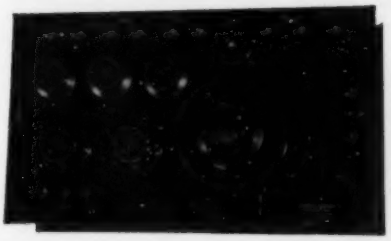

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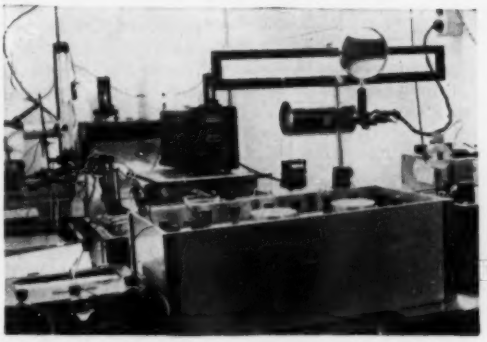

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